LANDAU FORTE ACADEMY QEMS

GCSE FOUNDATION

Mathematics

Knowledge Organiser

Name.....

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GCSE Mathematics Foundation
Algebra

DEFINITIONS

1	**Variable**	A letter representing a varying or unknown quantity	
2	**Coefficient**	A number which multiplies a variable.	e.g. 3 is the coefficient in 3y
3	**Term**	A part of an algebraic expression, could be a number, a variable or a product of both.	
4	**Like Terms**	Terms that have the same variable, but may have different coefficients	e.g. $a + 3a$ are like terms $b + a + a^2$ are not like terms
5	**Expression**	One or a group of terms. May include variables, constants, operators and grouping symbols. No '=' sign	
6	**Equation**	Expressions of equal value connected by an =	
7	**Identity**	An equation that is true no matter what values are chosen Uses '≡' sign	
8	**Formula**	A special type of equation that shows the relationship between different variables	
9	**Formulae**	Plural of Formula	
10	**Substitute**	Replace a variable with a number	
11	**Index**	A small number to the upper right of a base number that shows how many times the base is multiplied by itself	
12	**Power**	Another word for an index	
13	**Indices**	Plural of index	
14	**Expand**	Means multiply out one or more brackets	
15	**Factor**	A factor is a number **or variable** that divides into another number.	Eg. factors of 6x: 1, 2, 3, 6 & x
16	**Factorise**	Means writing an expression as a product of its factors The opposite of expanding out a bracket.	

ALGEBRAIC NOTATION

17	**Adding like terms**	Add the coefficients	e.g. $a + a = 2a$
18	**Subtracting like terms**	Subtract the coefficients	e.g. $5a - 3a = 2a$
19	**Multiplying like terms**	Add the powers	e.g. $a \times a = a^2$
20	**Dividing terms**	If the base is the same, subtract the powers	e.g. $a^5 \div a^2 = a^3$
21	**Adding different terms**	Cannot combine if the terms are different	e.g. $a + b = a + b$

22	**Subtracting different terms**	Cannot combine if the terms are different	e.g. $3a - 2b = 3a - 2b$
23	**Multiplying different terms**	Combine with no '×' sign	e.g. $a \times b = ab$
24	**Multiplying different terms with coefficients**	Combine with no '×' sign, multiply the coefficients	e.g. $3c \times 4a = 12ac$
25	**Dividing different terms**	Write as fractions with no '÷' sign	e.g. $3b \div a = \frac{3b}{a}$
26	**Dividing different terms with coefficients**	Write as fractions with no '÷' sign, simplify the coefficients if possible	e.g. $20c \div 4a = \frac{5c}{a}$

RULES OF INDICES

27	**Multiplying**	add the powers	$x^7 \times x^4 = x^{11}$
28	**Dividing**	subtract the powers	$x^9 \div x^3 = x^6$
29	**Brackets**	Multiply the powers	$(x^7)^2 = x^{14}$
30	**Power of 0**	Always = 1	$m^0 = 1$

EXPANDING SINGLE BRACKETS

31 Multiply all of the terms inside the bracket, by the term on the outside

32 $3(a + 4) = 3a + 12$

	$3x$	$+1$
5	$15x$	$+5$

$= 5x + 15$

EXPANDING DOUBLE BRACKETS

33 Multiply every term in the first bracket by every term in the second bracket

34

FOIL

e.g. $(x + 3)(x + 4)$

FIRST	x^2
OUTER	$+4x$
INNER	$+ 3x$
LAST	$+ 12$

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

GRID

e.g. $(x + 2)(x + 7)$

	x	+2
x	x^2	$+2x$
+7	$+7x$	$+14$

$= x^2 + 2x + 7x + 14$

$= x^2 + 9x + 14$

SMILEY FACE

e.g. $(x + 3)(x + 5)$

$(x + 3)(x + 5)$

$x^2 + 3x + 5x + 15$

$= x^2 + 8x + 15$

FACTORISING

35
- Find the highest common factor of the terms
- This goes outside the bracket
- Divide each term by the factor to get the new terms inside the bracket.

e.g. $2x + 6$

HCF is 2 $= 2(\quad)$

$2x \div 2 = x$

$6 \div 2 = 3$

$= 2(x + 3)$

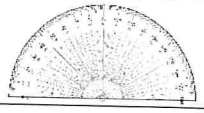
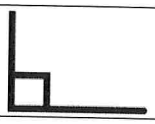
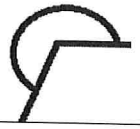
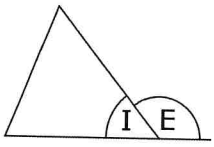
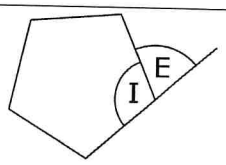
36
- CHECK by expanding your answer

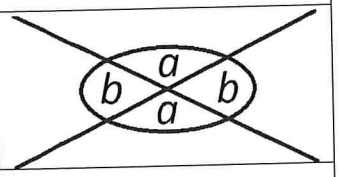
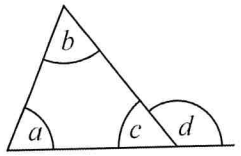
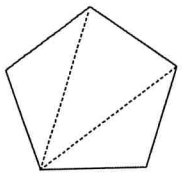
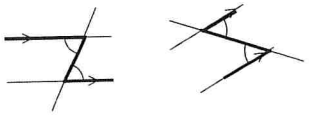
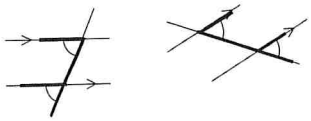
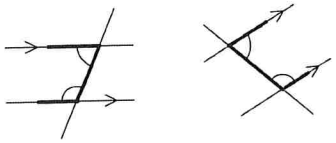
	x	+3
2	2x	+6

$= 2x + 6$ ✓

GCSE Mathematics Foundation
Angles

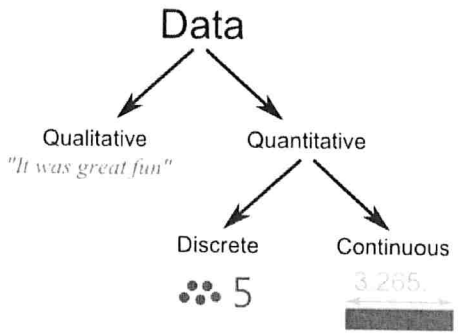
DEFINITIONS

1	**Protractor**	Equipment used to measure the size of an angle	
2	**Angle**	a measure of turn, measured in degrees ^{o}	
3	**Acute angle**	An angle less than 90^{o}	
4	**Right angle**	A 90^{o} angle	
5	**Obtuse angle**	An angle more than 90^{o} but less than 180^{o}	
6	**Reflex Angle**	An angle of more than 180^{o}	
7	**Parallel lines**		Equal distance apart If lines are extended, they will never meet.
8	**Interior angles (I)**	I is the interior angle which is inside the polygon	 
9	**Exterior angles (E)**	E is the exterior angle which is outside the polygon	For any polygon: $I + E = 180^{o}$
10	**Polygon**	A 2D shape with straight sides only	
11	**Regular Polygon**	All sides the same length All angles the same size	
12	**Congruent**	Two shapes are congruent when they are exactly the same shape and size	
13	**Similar**	Two shapes are similar when they are the same shape but different sizes	
14	**A Diagonal**	A line which joins opposite vertices of a shape	
15	**Bisect**	Cut in half	
16	**Tessellate**	Fit together leaving no gaps	

ANGLE RULES			
17	**Around a point**	Angles around a point add up to 360°	
18	**Straight line**	Angles on a straight line add up to 180°	
19	**In a triangle**	Angles in a triangle add up to 180°	
20	**Vertically Opposite**	Vertically opposite angles are equal	
ANGLES IN POLYGONS			
21	**Exterior angle in a triangle**	The exterior angle of a triangle equals the sum of the two opposite interior angles. angle d = angle a + angle b	
22	**Sum of interior angles**	For an n-sided polygon Sum of interior angles = 180 x (n – 2) The sum can also be found by splitting the polygon into triangles and multiplying the number of triangles by 180	
23	**Sum of exterior angles**	For all polygons: Sum of exterior angles = 360	
24	**Regular polygons**	E = 360 ÷ number of sides	
		An interior angle = sum of interior angles ÷ number of sides	
ANGLES ON PARALLEL LINES			
25	**Alternate angles**	Alternate angles on parallel lines are equal Alternate angles form a 'Z' shape	
26	**Corresponding angles**	Corresponding Angles on parallel lines are equal. Corresponding angles form an 'F' shape	
27	**Co-interior angles**	Co-interior angles add to 180 degrees. Co-interior angles form a 'C' or 'U' shape	

GCSE Mathematics Foundation
Averages and Range

DEFINITIONS

1	**Qualitative**	Data described by words	
2	**Quantitative**	Data that is categorized by numbers, it can be discrete or continuous	
3	**Discrete data**	Can be counted, can only have a finite number of possible values	
4	**Continuous data**	Can be measured, can have an infinite number of possible values within a selected range	
5	**Mean**	Add up all of the amounts. Divide by how many values there are	
6	**Median**	Put values in order. Locate the middle value	
7	**Mode**	The value that occurs most often	
8	**Range**	The biggest value minus the smallest value	
9	**Inequality signs**	< less than	> more than ; ≤ less than or equal to ; ≥ more than or equal to
10	**Compare**	Write two sentences comparing: a) an average (mean, mode or median) AND b) the range	
11	**Outlier**	An extreme data value that doesn't fit the overall pattern	
12	**Sample**	A selection from a whole group	
13	**Population**	The whole group	
14	**Bias**	Unfair, sway the results inaccurately	
15	**Random Sample**	Every member of the population has an equal chance of being included	

ADVANTAGES & DISADVANTAGES OF THE AVERAGES

Average	Advantages	Disadvantages
Mean	**Every value makes a difference**	**Affected by extreme values**
Median	**Not affected by extreme values**	**May not change if a data value changes**
Mode	**Easy to find; not affected by extreme values; can be used with non-numerical data**	**There may not be a mode**

AVERAGES FROM FREQUENCY TABLES

21 Modal Class

The class with the highest frequency

22 Median

If the total frequency is n, then the median lies in the class with the $\frac{n+1}{2}$th value in it.

23 Mean from a frequency table

Data value	Frequency	Frequency x Data Value
2	3	6
3	2	6
4	5	20
	10	32

Times →
Add ↓↓
Divide ←

Mean $= 32 \div 10 = 3.2$

24 Estimated mean from a grouped frequency table

Data value	Frequency	Midpoint	Frequency x Data Value
$20 \leq x < 30$	3	25	75
$30 \leq x < 40$	2	35	70
$40 \leq x < 50$	5	45	225
	10		370

Mean $= 370 \div 10 = 37$

There is a loss of accuracy due to using the midpoint, this is why it is only an estimate.

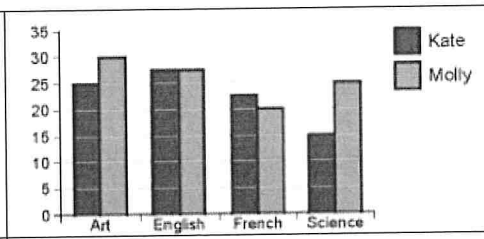
Times →
Add ↓↓
Divide ←

25 Estimate of range

The maximum possible value subtract the minimum possible value

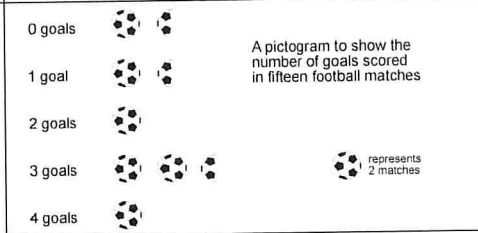
26 Dual bar chart

A bar chart to compare data sets by drawing adjoining bars for each.



27 Pictogram

A graph using pictures to represent quantity. Must have a key to say what each picture represents.



28 Stem and leaf

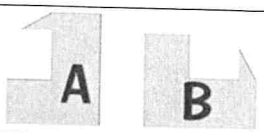
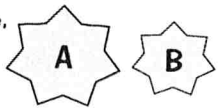
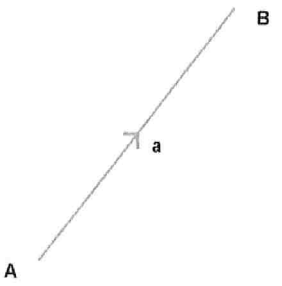
A data display that shows groups of data arranged by place value.
Leaves should be in order.
Must have a key.

STEM	LEAF
0	7
1	0 5 5 5 79
2	0 2 2 6 7
3	0 2 4 6 8

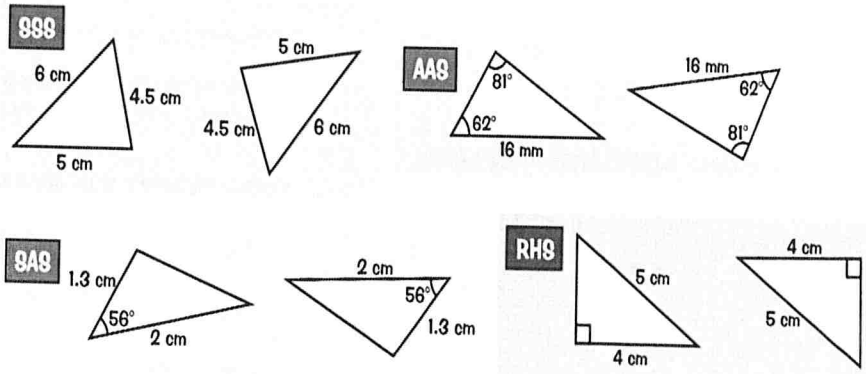
Key : 6 | 1 = 61 hours

GCSE Mathematics Foundation
Congruence, Similarity and Vectors

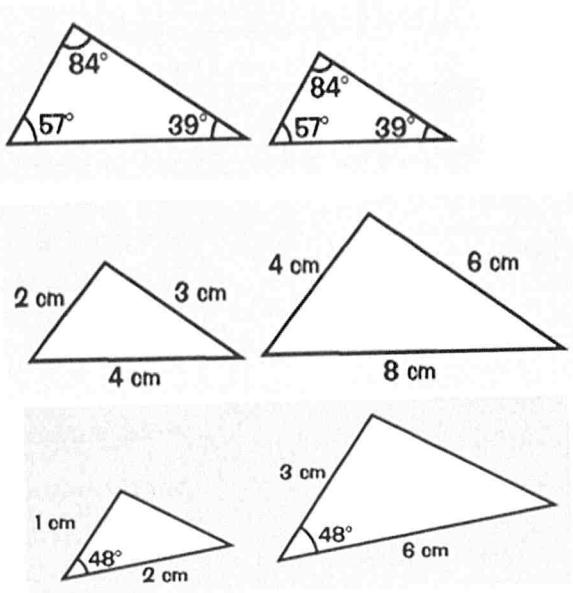
DEFINITIONS

1	**Congruence**	Same shape and same size	CONGRUENT — same size, same shape 
2	**Similar**	Same shape, different size, but all corresponding angles are equal	SIMILAR — same shape, different size 
3	**Vector**	**A vector describes a movement from one point to another.** **Vector notation** A **vector** quantity has both **direction** and **magnitude** (size). (In contrast a **scalar** quantity has **magnitude** only - eg, the numbers 1, 2, 3, 4...) 	For example this arrow represents a vector. The direction is given by the arrow, while the length of the line represents the magnitude. This vector can be written as: $\overrightarrow{AB}$, **a**, or $\begin{pmatrix}3\\4\end{pmatrix}$. In print, **a** is written in bold type. In handwriting, the vector is indicated by putting a squiggle underneath the letter: $\underset{\sim}{a}$
4	**Enlargement**	Same shape, different size, with all corresponding sides in the same ratio	An enlargement is **similar** to the original shape

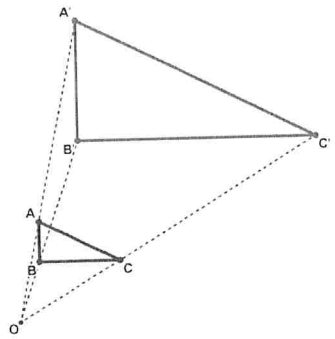
CONGRUENT TRIANGLES

5	**Two triangles are congruent if:** **a. 3 sides are the same (SSS)** **b. 2 angles and a corresponding side match (AAS)** **c. 2 sides and the angle between them match (SAS)** **d. A right angle, the hypotenuse and 1 other side all match (RHS)**	

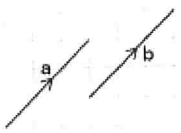
SIMILAR TRIANGLES

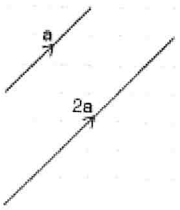
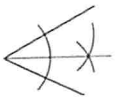
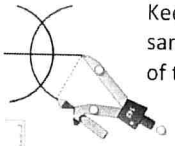
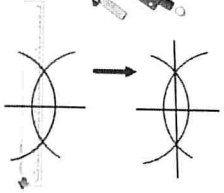
6	**2 triangles are similar, if a. all of the angles match; or** **b. all 3 sides are proportional; or** **c. any 2 sides are proportional and the angle between them is the same**	

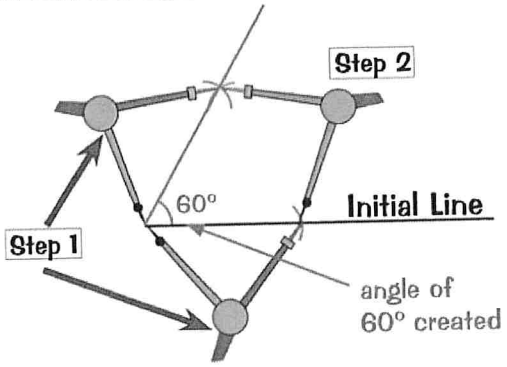
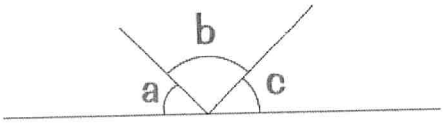
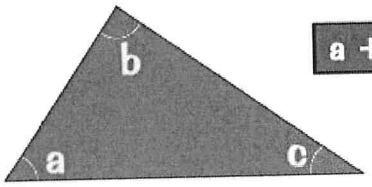
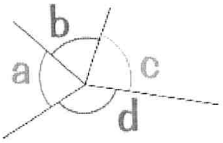
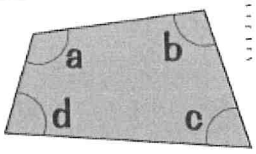
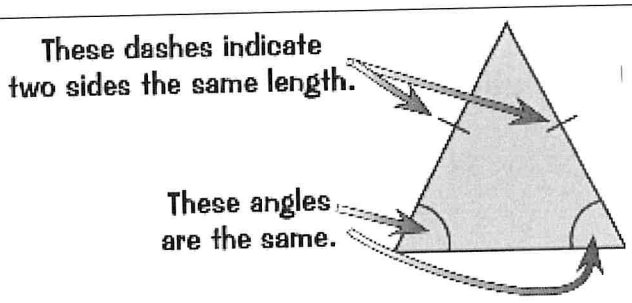
ENLARGEMENTS

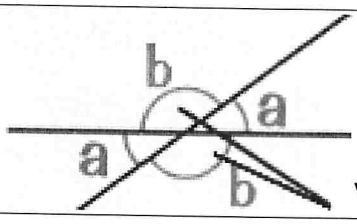
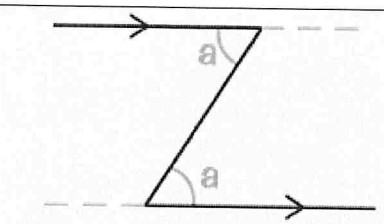
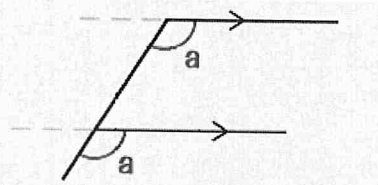
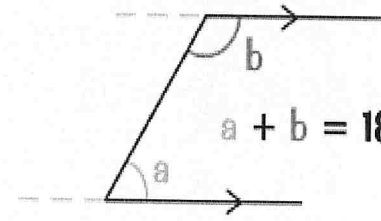
7	**Enlarging** a shape changes its size	2 parts to an enlargement • the scale factor • the centre of enlargement Fractional SF reduces the shape Negative SF inverts the shape An enlargement creates a similar shape	

VECTORS

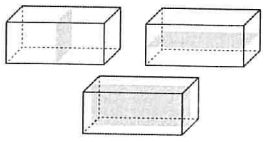
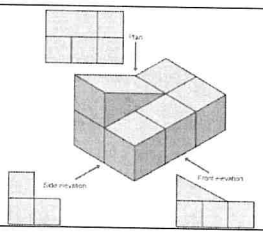
8	**Zero and Unit Vectors**	**Zero vector and unit vectors** A vector with magnitude 0 is called the zero vector, written **0**. A vector with magnitude 1 is called a unit vector. Vectors are equal if they have the same magnitude and the same direction. **a** = **b** 
9	**Inverse Vectors**	**Inverse Vectors** The inverse of a vector is a vector of equal magnitude but in the opposite direction. The inverse of $\overrightarrow{AB}$ is $-\overrightarrow{AB}$ or $\overrightarrow{BA}$ and the inverse of **a** is $-\mathbf{a}$.

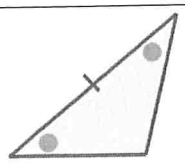
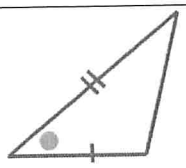
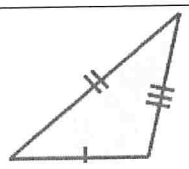
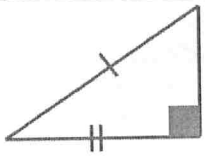
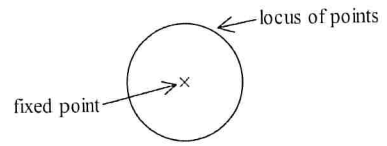
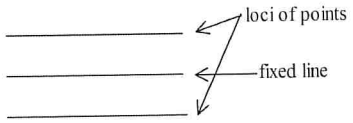
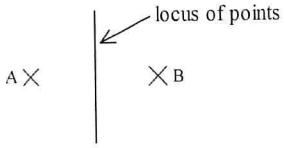
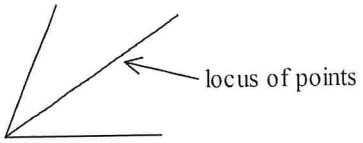
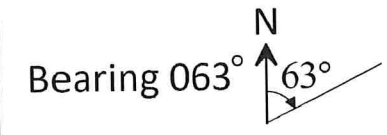
10	**Scalers**	**Scalars** Scalars are quantities which have magnitude but not direction. You can multiply a vector by a scalar to produce another vector. For example, multiplying vector **a** by **2** gives a vector twice the length of **a** and parallel to **a**. If $\mathbf{a} = \begin{pmatrix}3\\3\end{pmatrix}$ then $2\mathbf{a} = 2\begin{pmatrix}3\\3\end{pmatrix} = \begin{pmatrix}6\\6\end{pmatrix}$ 
CONSTRUCTIONS		
11	**Angle Bisector**	**Angle bisector** Place compass point on centre of angle. Open compass and draw an arc that cuts through both lines. Keep compass open to the same angle. Move the compass point to one of the points where the arc crossed the line. Draw an arc in the centre. Keep compass open to the same angle. Move the compass point to the other point where the arc crossed the line. Draw an arc in the centre.  Using a ruler draw a straight line through the centre of the angle to the point where the two arcs cross.
12	**Perpendicular Bisector**	**Perpendicular bisector** Place compass point on the end of the line and open to just over half way. Draw a semi-circle.  Keeping your compass open at the same angle, move it to the other end of the line. Draw another semi-circle.  Using your ruler draw a line through the two crosses that you have created.

13	**Constructing 60^0 angles**	 You can construct 30° angles and 45° angles by bisecting 60° and 90° angles
ANGLE FACTS		
14	**Angles on a straight line add up to 180**	**a + b + c = 180°** 
15	**Angles in a triangle add up to 180°**	 **a + b + c = 180°**
16	**Angles around a point add up to 360°**	 **a + b + c + d = 360°**
17	**Angles in a quadrilateral add up to 360°**	 **a + b + c + d = 360°**
18	**Base angles in an isosceles triangle are equal**	

19	**Vertically opposite angles are equal**	 Vertically opposite angles
PARALLEL LINE RULES		
20	**Alternate angles are equal**	 **Alternate angles are the same. They are found in a Z-shape.**
21	**Corresponding angles are equal**	 **Corresponding angles are the same. They are found in an F-shape.**
22	**Supplementary or allied angles add up to 180°**	

GCSE Mathematics Foundation
Constructions, Loci & Bearings

DEFINITIONS			
1	**Face**	A flat surface of a 3D shape (can be curved)	
2	**Edge**	A line segment where 2 faces meet	
3	**Vertex**	A point where 2 or more edges meet	
4	**Vertices**	Plural of vertex	
5	**Pyramid**	3D shapes which have a base of any shape, and sloping sides which meet at a point	
6	**Dimension**	A dimension is the size of something in a particular direction e.g. length, width, height, diameter	
7	**Plane**	A flat 2D surface	
8	**Plane of symmetry**	When a solid can be cut exactly in half, and a part on one side of the plane is an exact reflection of the part on the other side of the plane	
9	**Plan**	The view from above a solid	
10	**Front Elevation**	The view from the front of a solid	
11	**Side Elevation**	The view from the side of a solid	
12	**Scale**	A scale is a ratio that shows the relationship between a length on a drawing or a map and the actual length.	
13	**Construct**	Draw accurately using a ruler and a pair of compasses	
14	**Construction lines**	Working out lines / arcs are called construction lines	
15		Must not be rubbed out – they show the 'working out'	
16	**Cyclical Quadrilateral**	A four sided polygon whose vertices are all on the circumference of a circle	
17	**Bisect**	Cut in half	
18	**Perpendicular**	At right angles	
19	**Perpendicular bisector**	The line that cuts another in half at right angles	
20	**Angle bisector**	The line that cuts an angle exactly in half	
21	**Locus**	The set of all points that obey a certain rule. This is often drawn as a continuous path	
22	**Loci**	The plural of locus	
23	**Region**	An area bounded by loci	

DRAWING ACCURATE TRIANGLES			
24	**ASA**	Given angle, side, angle you can draw an accurate triangle	
25	**SAS**	Given two sides and the angle in between you can draw an accurate triangle	
26	**SSS**	Given all three sides you can draw an accurate triangle	
27	**RHS**	Given that it has a right angle, the hypotenuse and another side length, you can draw an accurate triangle	
LOCI			
28	**Locus 1: Circle**	The locus of points that are a fixed distance from a fixed point	
29	**Locus 2: parallel line**	The locus of points a fixed distance from a fixed line	
30	**Locus 3: Perpendicular bisector**	The line that cuts another in half at right angles	
31	**Locus 4: angle bisector**	The locus of points equidistant between two fixed points	
BEARING			
32	**Bearing**	Angle measured clockwise	
33		Measured from north	
34		Always written using 3 digits	

Year 9 GCSE Mathematics Foundation
Equations, Inequalities & Sequences

DEFINITIONS

1	**Integer**	Whole number	
2	**Term**	One part of an algebraic expression which may be a number, a variable or a product of both.	e.g. 3x
3	**Expression**	An expression is one or a group of terms and may include variables, constants, operators and grouping symbols	e.g. 5x + 2y
4	**Equation**	A mathematical statement containing an equals sign (=), to show that two expressions are equal.	e.g. 3x + 4 = x - 2
5	**Formula**	A special type of equation that shows the relationship between different variables	
6	**Identity**	An equation which is always true regardless of the value of the variables. Has an identity sign (≡)	e.g. $3(x + 5) \equiv 3x + 15$
7	**Unknown**	A letter representing a number	
8	**Subject**	The subject of a formula is the variable on its own on one side of the equals sign	
9	**Inverse Operation**	The operation that reverses the effect of another operation.	
10	**Solve**	To find the value of the unknown	
11	**Inequality**	$>$ Is greater than	$\geq$ Is greater than or equal to
12		$<$ Is less than	$\leq$ Is less than or equal to
13	**Inclusive**	Gives a finite range of solutions	e.g. $3 \leq x < 7$
14	**Exclusive**	Gives an infinite range of solutions	e.g $4 < x$ $x < 1$
15	**Sequence**	an ordered pattern of numbers or diagrams	
16	**Term**	One of the numbers in a sequence	

SOLVING EQUATIONS

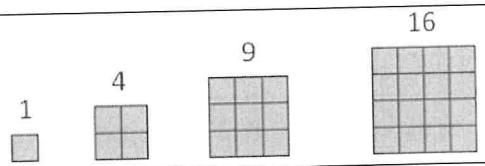
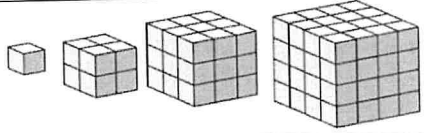
17	To solve an equation we use inverse operations	
18	What ever you do to one side of the equation, you have to do to the other side to keep it "balanced" and equal	
19	**One step**	Solving equations – one step

$$\begin{array}{rcl} x & = & 7 \\ +4 & & \\ (-4) & & (-4) \\ x & = & 11 \end{array} \qquad \begin{array}{rcl} x & = & 12 \\ -5 & & \\ (+5) & & (+5) \\ x & = & 17 \end{array} \qquad \begin{array}{rcl} 3x & = & 18 \\ & & \\ (\div 3) & & (\div 3) \\ x & = & 1 \end{array} \qquad \begin{array}{rcl} \frac{x}{4} & = & 6 \\ & & \\ (\times 4) & & (\times 4) \\ x & = & 24 \end{array}$$

20	**Two step**	Requires two inverse operations to solve	$2x - 7 = 19$ $2x = 26$ $x = 13$
21	**Involving brackets**	Expand the brackets first	$5(2x + 1) = 35$ $10x + 5 = 35$ $10x = 30$ $x = 3$
22	**Unknowns both sides**	Eliminate the x term from one of the sides	$5x + 2 = 3x - 8$ $2x + 2 = -8$ $2x = -10$ $x = -5$

INEQUALTITIES			
23	**Solve**	Inequalities are solved using the same steps as equations.	
24	**List integer solutions**	Give the integers that satisfy the inequality e.g. $x > 5$ integer solutions are 6, 7, 8 ..... e.g. $3 \leq x < 7.4$ integer solutions are 3, 4, 5, 6, 7	
25	**Represent on a number line**	An empty circle shows the value is NOT included	
		A full circle shows the value IS included	
		An arrow shows the solution continues to infinity	
26	**Negative multiples**	If you multiply or divide an inequality by a negative number, then the inequality sign is reversed	

SEQUENCES			
27	**Term-to-term rule**	the rule for moving from one number in a sequence to the next number	e.g. 4 7 10 ..... Term-to-term rule is + 3
28	**Arithmetic sequence**	a sequence where the term to term rule is to add or subtract the same amount each time	e.g. 1 5 9 ..... 4 is added each time
29	**Geometric sequence**	A sequence where the term to term rule is to multiply by the same amount each time	e.g. 2 20 200 ..... x10 each time
30	**Position-to-term rule (nth term)**	An expression that that can be used to calculate any term in a sequence given its position (n) in the sequence.	e.g. 4 7 10 13 16..... 3n + 1

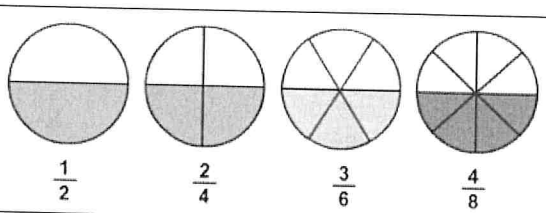
MUST KNOW SEQUENCES			
31	**Square numbers**	1, 4, 9, 16, 25, 36 ...	
32	**Cube numbers**	1, 8, 27, 64, 125	
33	**Triangular numbers**	1, 3, 6, 10, 15, 21, 28	O O O O O OO OO OO OO OOO OOO OOO OOOO OOOO OOOOO

GCSE Mathematics Foundation
Fractions & Percentages

DEFINITIONS

1	**Fraction**	Part of a whole	
2	**Equivalent Fractions**	Fractions which have the same value but look different	e.g. $\frac{1}{2} = \frac{3}{6} = \frac{10}{20}$
3	**Simplifying Fractions**	Divide numerator and denominator by HCF	e.g. $\frac{24}{30} = \frac{4}{5}$
4	**Fraction of an Amount**	Amount divided by the denominator then multiplied by the numerator.	e.g. $\frac{5}{7}$ of 42 $42 \div 7 \times 5 = 30$
5	**Mixed Number**	A number made from integer and fraction parts	e.g. $1\frac{3}{8}$
6		Before multiplying, dividing, adding or subtracting, always change mixed numbers into Improper fractions	
7	**Improper Fraction**	A number made from integer and fraction parts	e.g. $1\frac{3}{8}$
8	**Compare**	To compare fractions, write them with a common denominator	
9	**Unit Fraction**	Has a numerator of 1	e.g. $\frac{1}{6}$
10	**Reciprocal**	The reciprocal of a number is one divided by the number.	e.g. reciprocal of 7 is $\frac{1}{7}$
11		Dividing by a number is the same as multiplying by its reciprocal	e.g. $\div 2$ is the same as $\times \frac{1}{2}$
12	**Terminating Decimal**	Decimals that can be written exactly.	0.38
13	**Recurring Decimal**	Decimals where one digit or a group of digits are repeated.	0.777..... $= 0.\dot{7}$ 0.803803.... $= 0.\dot{8}0\dot{3}$
14	**Simple Interest**	Interest is calculated as a percent of an original loan	

FRACTIONS

15	**Equivalent fractions**	
16	To find equivalent fractions – multiply the numerator (top) and the denominator (bottom) by the same number. Here multiplying both by 5 $\frac{1}{2} = \frac{5}{10}$	
17	To cancel fractions – divide the numerator (top) and the denominator (bottom) by the same number. Here dividing both by 4 $\frac{12}{28} = \frac{3}{7}$	

18	**Multiply Fractions**	Multiply the numerators and multiply the denominators.	$\frac{9}{7} \times \frac{4}{5} = \frac{9\times4}{7\times5} = \frac{36}{35}$
19	**Divide Fractions**	Flip the second fraction (find the reciprocal). Change the divide to a multiply. Multiply the fractions	$\frac{9}{7} \div \frac{5}{6} = \frac{9}{7} \times \frac{6}{5} = \frac{9\times6}{7\times5} = \frac{54}{35}$
20	**Add or Subtract Fractions**	Write as fractions with a common denominator. Add or subtract the numerators.	$\frac{2}{8} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24} = \frac{10}{24} = \frac{5}{12}$

PERCENTAGES			
21	**Percentage**	Means "out of 100"	
22	**Percentage Increase**	Calculate the percentage and add it on to the original	
		Or use a multiplier	$\frac{100 + \%\ increase}{100}$
23	**Percentage decrease**	Calculate the percentage and subtract it from the original	
		Or use a multiplier	$\frac{100 - \%\ increase}{100}$

CONVERSIONS			
24	**Percentage to Decimal**	Divide by 100	
25	**Decimal to Percentage**	Multiply by 100	
26	**Fraction to Percentage**	Find an equivalent fraction with 100 as the denominator	
27	**Percentage to Fraction**	Write as a fraction over 100 and simplify	
28	**Fraction to Decimal**	Division or convert to a % first	
29	**Decimal to Fraction**	Use place value to find the denominator and simplify	
	FRACTION	**DECIMAL**	**PERCENTAGE**
30	$\frac{1}{4}$	0.25	25%
31	$\frac{1}{2}$	0.5	50%
32	$\frac{3}{4}$	0.75	75%
33	$\frac{1}{10}$	0.1	10%
34	$\frac{1}{5}$	0.2	20%
35	$\frac{1}{3}$	$0.\dot{3}$	$33.\dot{3}\%$
36	$\frac{2}{3}$	$0.\dot{6}$	$66.\dot{6}\%$
37	$\frac{1}{8}$	0.125	12.5%

GCSE Mathematics Foundation
Fractions, Indices and Standard Form

DEFINITIONS

1	**Mixed Number**	A number made from integer and fraction parts	e.g. $1\frac{3}{8}$
		Before multiplying, dividing, adding or subtracting, always change mixed numbers into improper fractions	
2	**Improper Fraction**	A number made from integer and fraction parts	e.g. $1\frac{3}{8}$
3	**Powers of 10**	$10^1 = 10$ $10^2 = 100$ $10^3 = 1000$ $10^4 = 10\,000$	$10^5 = 100\,000$ $10^6 = 1000\,000$ $10^7 = 10\,000\,000$ $10^8 = 100\,000\,000$
4	**Indices**	**Indices** are the numbers written above a base number. E.g. For 8^2, 8 is the base number and 2 is the index The **index** shows how many times it has been multiplied by itself. E.g. $3^7 = 3x3x3x3x3x3x3$	
5	**Reciprocal**	The reciprocal of a number is 1 ÷ the number	The reciprocal of 5 is $\frac{1}{5}$. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$
6	**Standard Form**	A number is converted into **standard form** when the number is very large or very small. The format of a number in standard form consists of a number between 1 and 10, **but cannot be 10,** multiplied by a power of 10. E.g. 147 100 000 000 = 1.471×10^{11}	

FRACTIONS

7	**Multiply Fractions**	Multiply the numerators and multiply the denominators.	$\frac{9}{7} \times \frac{4}{5} = \frac{9\times4}{7\times5} = \frac{36}{35}$
8	**Divide Fractions**	Flip the second fraction (find the reciprocal). Change the divide to a multiply. Multiply the fractions	$\frac{9}{7} \div \frac{5}{6} = \frac{9}{7} \times \frac{6}{5} = \frac{9\times6}{7\times5} = \frac{54}{35}$
9	**Add or Subtract Fractions**	Write as fractions with a common denominator. Add or subtract the numerators.	$\frac{2}{8} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24} = \frac{10}{24} = \frac{5}{12}$
10	**Convert between Improper and Mixed fractions**	Divide the numerator by the denominator. The answer gives the whole number part. The remainder goes on top of the fraction, with the same denominator.	$\frac{43}{6} = 7\frac{1}{6}$

11	**Convert between Mixed and Improper fractions**	Multiply the denominator by the whole number part and add the numerator. Put the answer over the numerator.	$7\frac{1}{6} = (6x7+1)/6 = \frac{43}{6}$
INDICES			
12	**Index Laws**	When multiplying, we add the powers.	$2^3 \times 2^5 = 2^{3+5} = 2^8$ $a^3 \times a^5 = a^{3+5} = a^8$
		When dividing, we subtract the powers.	$5^7 \div 5^2 = 5^{7-2} = 5^5$ $b^7 \div b^2 = b^{7-2} = b^5$
		The above laws don't work for different bases: $2^3 \times 3^7$	
		When raising one power to another, we multiply the powers	$(3^4)^5 = 3^{4x5} = 3^{20}$ $(c^4)^5 = c^{4x5} = c^{20}$
13		Anything to the power of 1 is just itself	$5^1 = 5$; $x^1 = x$
14		Anything to the power of 0 is 1	$5^0 = 1$; $x^0 = 1$
STANDARD FORM			
15	**Converting a very small number into standard form**	$0.00000037 = 3.7 \times 10^{-7}$	
16	**Converting a very large number into standard form**	$147\ 100\ 000\ 000 = 1.471 \times 10^{11}$	
17	**Converting into a small ordinary number**	$2.4 \times 10^{-6} = 0.0000024$	
18	**Converting into a large ordinary number**	$5.67 \times 109 = 5\ 670\ 000\ 000$	
19	**When adding or subtracting numbers in standard form**	The numbers must be converted into the ordinary numbers $(2.3 \times 10^4) + (6.4 \times 10^3) =$ $23000 + 6400 =$ $29400 = 2.94 \times 10^4$	
20	**When multiplying numbers in standard form**	The format stays the same. We can use **index laws** to help us. $(1.5 \times 10^3) \times (3 \times 10^5) =$ $4.5 \times 10^{3+5} =$ 4.5×10^8	
21	**When dividing numbers in standard form**	The format stays the same. We can use **index laws** to help us. $(2.5 \times 10^{11}) \div (5 \times 10^{13}) =$ $0.5 \times 10^{-2} =$ 5×10^{-3}	

GCSE Mathematics Foundation
Graphs Tables & Charts

DEFINITIONS

1	**Qualitative**	Data described by words			
2	**Quantitative**	Data that is categorized by numbers, it can be discrete or continuous			
3	**Discrete data**	Can be counted, can only have a finite number of possible values			
4	**Continuous data**	Can be measured, can have an infinite number of possible values within a selected range			
5	**Inequality signs**	< less than	> greater than	≤ less than or equal to	≥ greater than or equal to

DATA DISPLAYS

6	Two Way Tables	Tables used to compare the relationship between two discrete data categories.	

	Walk	Car	Other	Total
Boy	15		14	54
Girl		8	16	
Total	37			100

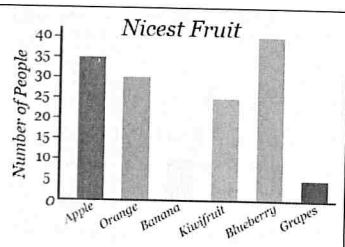
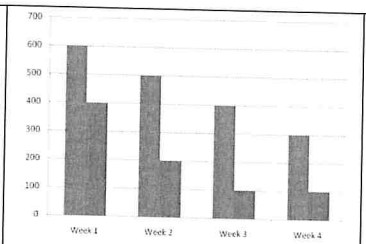
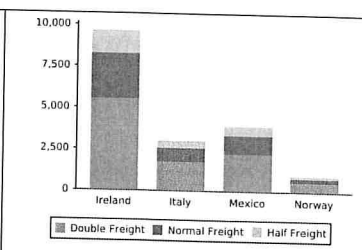
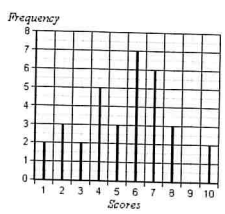
7	Distance Chart	A convenient way of showing the distances between several places		

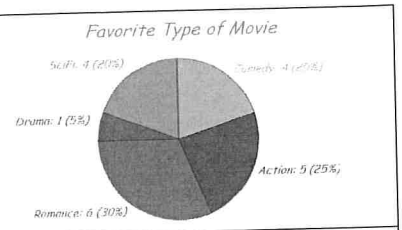
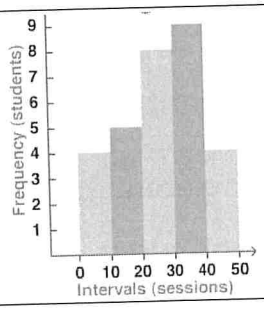
Bristol				
45	Cardiff			
205	230	Dover		
85	110	245	Exeter	
230	250	265	305	Hull

8	Stem and leaf	A data display that shows groups of data arranged by place value	
9		A back – to – back stem & leaf diagram compares 2 sets of results	
10		Must always have a key	

A		B
LEAF	STEM	LEAF
8 8 7 5	0	7
9 7 4 1 0	1	0 5 5 5 7 9
2 2 2 1	2	0 2 2 6 7
8 6 4 2 0	3	0 2 4 6 8

Key : 6 | 1 = 61 hours

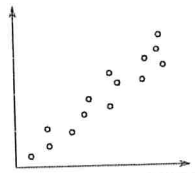
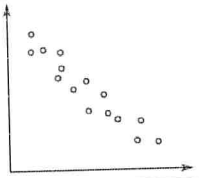
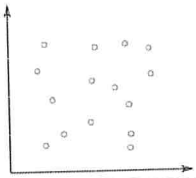
11	**Bar Chart**	A graphical display of data where the bar heights show the frequencies	
12			 Comparative Bar Chart
			 Composite Bar Chart
13	**Time Series Graph**	A line graph with time plotted on the horizontal axis	
14	**Vertical Line Diagram**	A graphical display of data where the line heights show the frequencies	

15	**Pie Chart**	A circular chart where the pie sectors are used to show the relative sizes of data	
16	**Histogram**	Plotted for grouped continuous data	
17		No gaps between the bars	

SCATTER DIAGRAMS

18	**Scatter diagram**	A graphical diagram with points plotted to show a relationship between two variables
19	**Bivariate data**	Data of two variables we want to compare to look for relationships

20

Positive Correlation	**Negative Correlation**	**No Correlation**
		
As one variable increases, the other variable increases	As one variable increases, the other variable decreases	As one variable increases, the other variable shows no connection

21	**Line of best fit**	A line of best fit is drawn on a scatter diagram so that it is as close as possible to the points
22	**Interpolation**	Using a line of best fit to predict values within the range of data given. Usually accurate
23	**Extrapolation**	Using a line of best fit to predict values outside the range of data given. May not be accurate
24	**Outlier**	A value far away from the rest of the data

FREQUENCY TABLES

25

Qualitative

Method of Travel	*Tally*	*Frequency*
Walk	𝍸 IIII	9
Bike	III	3
Car	𝍸 I	6
Bus	𝍸 𝍸 II	12
	TOTAL	30

Quantitative

score	tally	frequency (f)
1	IIII	4
2	𝍸 IIII	9
3	𝍸 I	6
4	𝍸 II	7
5	III	3
6	II	2

Grouped - Discrete

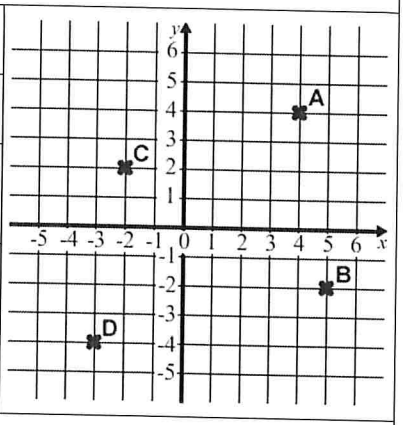
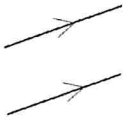
Class (Marks)	Frequency
11 - 15	2
16 - 20	3
21 - 25	3
26 - 30	5
31 - 35	6
36 - 40	6
41 - 45	3
46 - 50	2
Total	30

Grouped – Continuous

Weight of box (w kg)	Frequency
$0 < w \leqslant 4$	11
$4 < w \leqslant 8$	16
$8 < w \leqslant 12$	29
$12 < w \leqslant 16$	26
$16 < w \leqslant 20$	20

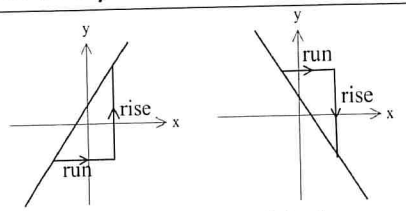
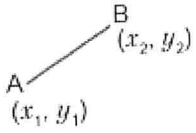
GCSE Mathematics Foundation
Graphs

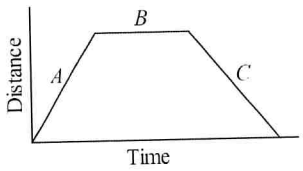
DEFINITIONS

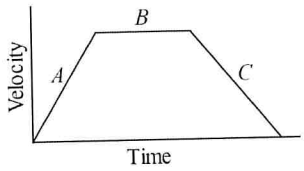
1	**Axis**	A reference line on a graph	
2	**Axes**	Plural of axis	
4	**Quadrant**	A quarter of a graph separated by axes	
5	**Origin**	The point (0, 0) on a set of axes	
3	**Coordinates**	Used to show a position on a coordinate plane	
		The first coordinate is the horizontal position (x axis), the second is the vertical position (y axis)	
6	**Parallel**		Equal distance apart If lines are extended, they will never meet.
7	**'y =' graph**	Constant y co-ordinate	
		Will be parallel to the x-axis	
8 9	**'x =' graph**	Constant x co-ordinate	
		Will be parallel to the y-axis	
10	**y-intercept**	The y value where a graph crossed the y – axis.	where x = 0
11	**Gradient**	The steepness of a graph	
12	**Line segment**	A part of a line, has a start point, and an end point	
13	**Labelling a graph**	Means write the equation next to the line	
14	**Coefficient**	Number in front of a variable	
15	**Linear Equation**	Produces a straight line graph	
16	**Steady speed**	Means travelling the same distance each minute	
17	**Velocity**	Means speed in a certain direction	
18	**Rate of Change Graph**	Shows how quantity changes over time	

STRAIGHT LINE GRAPHS

19	**Linear Equation**	A linear equation contains an x term but no higher power of x.	e.g. $y = 3x - 1$ $2x + 3y = 8$

20	**y = mx + c**	The general equation of a straight line	m is the gradient c is the y-intercept
21	**Gradient**	$Gradient = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$	 This has a positive gradient This has a negative gradient
22		Positive gradients, line goes from bottom left to top right	
23		Negative gradients, line goes from top left to bottom right	
24	**Gradient between 2 points**	If A = (x_1, y_1) and B = (x_2, y_2) The gradient of line AB = $\frac{y_2-y_1}{x_2-x_1}$	
25	**Parallel lines**	Parallel lines have the same gradient	
26	**Mid-point**	The mid-point is the coordinate half between two points.	If A = (x_1, y_1) and B = (x_2, y_2) the mid-point is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

DISTANCE – TIME GRAPHS		
27	Represent a journey	 A = steady speed, B = no movement, C = steady speed back to start
28	The vertical axis represents the distance from a starting point	
29	The horizontal axis represents time taken	
30	Straight lines mean constant speed	
31	Horizontal lines mean no movement	
32	Gradient = speed	
33	$Average\ speed = \frac{Total\ distance}{Total\ time}$	

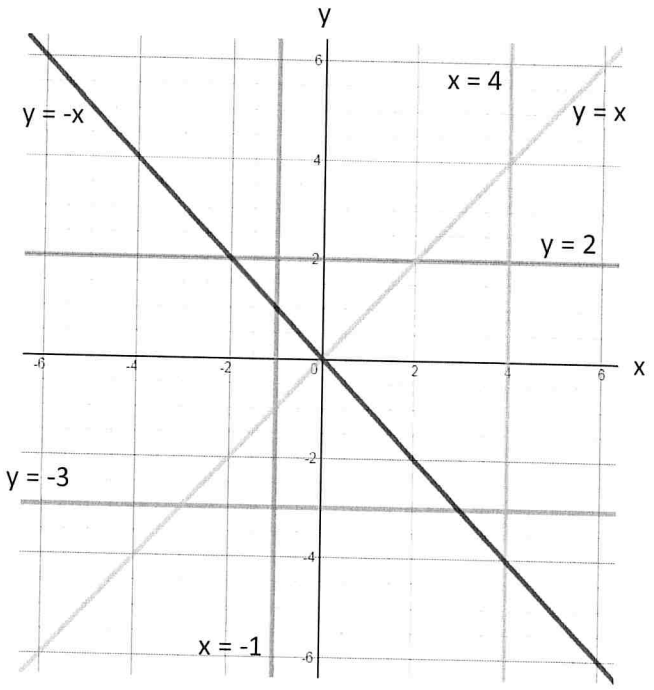
VELOCITY – TIME GRAPHS		
34	Represent the speed at a given time	 A = steady acceleration, B = constant speed, C = steady deceleration back to a stop
35	Straight lines mean constant acceleration/deceleration	
36	Horizontal lines mean no change in velocity (speed)	
37	Positive Gradient = acceleration	
38	Negative Gradient = deceleration	
39	The area under the graph = distance travelled	

GCSE Mathematics Foundation
More Algebra

DEFINITIONS

1	**Linear**	Going up in equal amounts	Linear graphs are straight line graphs
2	**Coordinates**	Points plotted on a set of (x, y) axes	(3,4) is 3 points in the x (horizontal) direction and 4 points in the y (vertical) direction
3	**Function**	A relation or expression involving one or more variables	$f(x) = ax^2 + bx + c$
4	**Simultaneous Equations**	Two equations in two variables having the same solutions	$3x + 2y = 17$ $4x - 2y = 18$
5	**Quadratic**	Of a square power	$y = x^2 + 2x + 3$
6	**Cubic**	Of a cube power	$Y = x^3 - 4x$
7	**Reciprocal**	The reciprocal of a number is 1 divided by that number. A number multiplied by its reciprocal, will be equal to 1	The reciprocal of 3 is $\frac{1}{3}$

STRAIGHT LINE GRAPHS

8	**For any equation of the form x = a, where a can take any value, the line will be vertical.** **For any equation of the form y = b, where b can take any value, the line will be horizontal.** **y = x is a diagonal line.**	

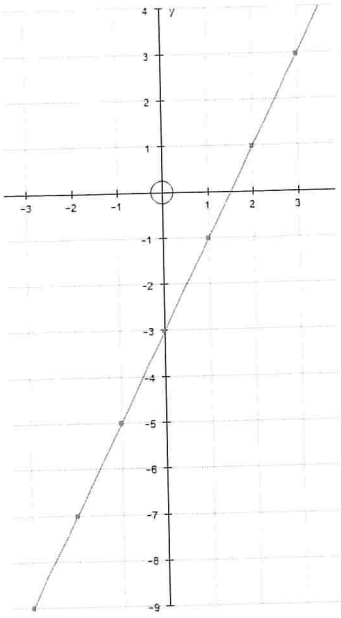
Example:

Plot the graph of $y = 2x - 3$ for $-3 \leq x \leq 3$.

- First draw a table of values and substitute each x-value into the formula to find the corresponding y-value.

x	-3	-2	-1	0	1	2	3
y	-9	-7	-5	-3	-1	1	3

- Plot each pair of coordinates: (-3, -9), (-2, -7), etc. and draw a straight line through all of the points that reaches across the coordinate grid.



SOLVING LINEAR EQUATIONS

9

To solve an equation, you find the value of the unknown by isolating it on one side by performing inverse operations. Remember that solutions can be integers, decimals, fractions and negative numbers.

Solve	$f - 5 = -1$
Add 5 to both sides	$+5 \quad +5$
	$f = 4$

Solve	$4y + 1 = 9$
Subtract 1	$-1 \quad -1$
	$4y = 8$
Divide by 4	$\div 4 \quad \div 4$
	$y = 2$

Solve	$3(4m + 1) = 15$
Expand bracket	$12m + 3 = 15$
Subtract 3	$-3 \quad -3$
	$12m = 12$
Divide by 12	$\div 12 \quad \div 12$
	$m = 1$

Solve	$5e - 1 = 3e + 6$
Subtract 3e	$-3e \quad -3e$
	$2e - 1 = 6$
Add 1	$+1 \quad +1$
	$2e = 7$
Divide by 2	$\div 2 \quad \div 2$
	$e = 3.5$

SOLVING LINEAR SIMULTANEOUS EQUATIONS

10

This is when you have two equations with two unknowns which you can solve either algebraically or graphically.

Example 1

Solve the simultaneous equations:

$$3x + y = 7\ (1)$$
$$5x + y = 5\ (2)$$

Label the equations 1 and 2.

The coefficient of the y values is the same so we want to eliminate the y values. We must subtract one equation from the other as both y values have the same sign.

(2) – (1)

$$\begin{array}{rl} 5x + y = 5 & - \\ 3x + y = 7 & \\ \hline 2x \quad = -2 & \end{array}$$

Therefore, $x = -1$.

Substitute this into equation (1)

$$3 \times -1 + y = 7$$
$$-3 + y = 7$$
$$y = 10$$

Check your answers by substituting both values into the other equation.

$$5 \times -1 + 10 = 5$$

Example 2

Solve the simultaneous equations:

$$5x - 2y = 25\ (1)$$
$$4x + 3y = 43(2)$$

In this example, the coefficients of x and of y are different. We must start by finding the LCM of either the x or the y coefficients and multiplying both equations to get the same coefficient.

The LCM of 2 and 3 (the y coefficients) is 6.

(1) x 3 $\quad 15x - 6y = 75\ (3)$

(2) x 2 $\quad 8x + 6y = 86\ (4)$

Now that the coefficients of the y values are the same we can eliminate the y values. We must add the equations as the signs of the y values are different.

(3) + (4)

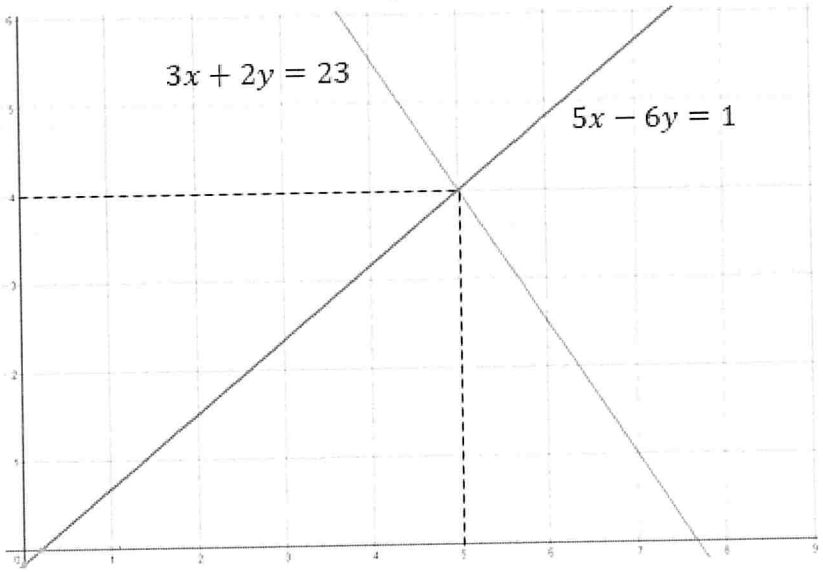
$$\begin{array}{rl} 15x - 6y = 75 & \\ 8x + 6y = 86 & + \\ \hline 23x \quad = 161 & \end{array}$$

Therefore, $x = 7$.

Substitute this into equation (1)

$$5 \times 7 - 2y = 25$$
$$35 - 2y = 25$$
$$2y = 10$$
$$y = 5$$

Check in equation (2) as before.

		Example 3 – Graphically Solve the simultaneous equations: $3x + 2y = 23$ $5x - 6y = 1$  The solution to these simultaneous equations is given by the point where the lines meet. The x-coordinate gives the solution of x and the y-coordinate gives the solution of y. In this example, the solution is $x = 5$ and $y = 4$.
Recognising and Sketching Graphs		
11	**Quadratic Graph**	The highest power of x is 2. This graph shows $y = x^2$. Other examples of quadratic graphs could be $y = x^2 + 2$, $y = 3x^2$, or $y = x^2 + 3x - 4$.
12	**Cubic Graph**	The highest power of x is 3. This graph shows $y = x^3$. Other examples of cubic graphs could be $y = x^3 - 5$, $y = 2x^3$, or $y = x^3 + x^2 + x + 1$.
13	**Reciprocal Graph**	This graph shows $\frac{1}{x}$. In this example, there are asymptotes at the x and y axes – this means that the graph gets closer and closer to the axes without ever touching them.

GCSE Mathematics Foundation
Multiplicative Reasoning

DEFINITIONS

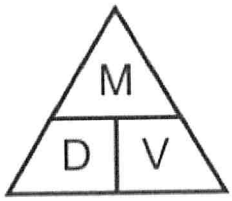
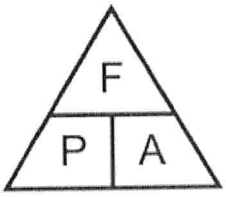
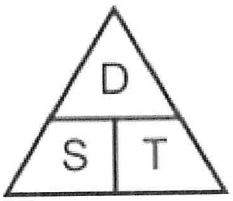
1	**Annual**	Means yearly	
2	**Per annum**	Means per year	
3	**Salary**	A fixed regular payment, often paid monthly	
4	**Simple Interest**	Interest is calculated as a percent of an original loan	
5	**Compound Interest**	When interest is calculated on both the amount borrowed AND any previous interest	
6	**Half - Life**	The time taken for the count rate to fall to half its starting value	
7	**Tax**	A financial charge placed on sales or savings by the government e.g. VAT	
8	**Loss**	Income minus all expenses, resulting in a negative value.	
9	**Profit**	Income minus all expenses, resulting in a positive value.	
10	**Compound Measures**	Combine measures of two different quantities	
11	**Velocity**	Speed in a given direction	Usually measured in m/s
12	**Acceleration**	The rate of change of velocity	Usually measured in m/s^2
13	**Rest**	If something "starts from rest" it means its initial velocity is zero	

PERCENTAGES

14	The **original amount** is always 100%	If you increase, the new amount will be more than 100%
		If you decrease, the new amount will be less than 100%
15	**Express one number as a percentage of another**	$\frac{Number\ 1}{Number\ 2} \times 100$
16	**Percentage change**	$\frac{Change}{Original} \times 100$
17	**To find a multiplier for an increase**	$\frac{100 + \%\ increase}{100}$
18	**To find a multiplier for a decrease**	$\frac{100 - \%\ decrease}{100}$

COMPOUND INTEREST FORMULA

19	$P\left(1 + \frac{R}{100}\right)^n$	P – Original amount R – Interest rate n – the number of interest periods (e.g. years)
20	**or** $Original \times Multiplier^{time\ period}$	

21	**Density**	$density = \frac{mass}{volume}$	
22		The mass of a substance contained in a certain volume	
23		Usually measured in g/cm^3 or kg/m^3	
24	**Pressure**	$pressure = \frac{force}{area}$	
25		The force applied over an area	
26		Usually measured in N/m^2	
27	**Speed**	$speed = \frac{distance}{time}$	
28		The distance travelled in an amount of time	
29		Usually measured in m/s (metres per second) or km/h (kilometres per hour) or mph (miles per hour)	

KINEMATICS

30	$v = u + at$	*a* is constant acceleration *u* is initial velocity *v* is final velocity *t* is the time taken *s* is the displacement from the position when *t*=0
31	$s = ut + \frac{1}{2}at^2$	
32	$v^2 = u^2 + 2as$	

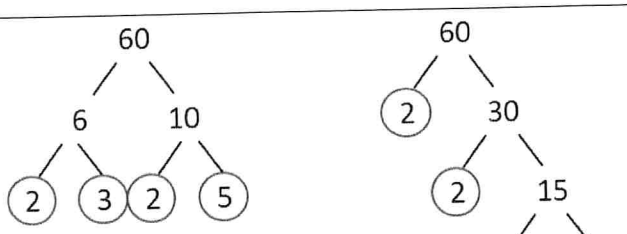
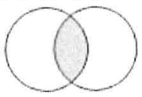
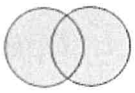
PROPORTION

33	**Direct proportion**	Two quantities increase at the same rate	"*y* is proportional to *x*" $y \propto x$ $y = kx$
34		Pairs of values will be in the same ratio	
35	**Indirect Proportion**	One quantity increase at the same rate as the other quantity decreases	"*y* is inversely proportional to *x*" $y \propto \frac{1}{x}$ $y = \frac{k}{x}$
36	**Constant of proportionality**	Represented by a *k*	Its value stays the same

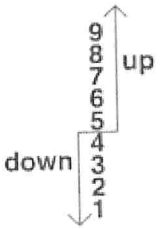
GCSE Mathematics Foundation
Number

DEFINITIONS

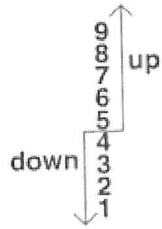
2	**Integer**	a whole number							
3	**Positive**	A number greater than zero							
4	**Negative**	A number less than zero							
5	**Decimal**	A number with digits after the decimal point							
6	**Surd**	A number written exactly using a square root							
7	**Operations**	Symbols and words used to show how to combine numbers							
		×	Multiply	+	Add	÷	Divide	–	Subtract
8	**Inverse Operations**	The operation used to reverse the original operation.							
		e.g. + and -				e.g. × and ÷			
9		e.g. Finding a square root is the inverse of finding the square							
10		e.g. Finding a cube root is the inverse of finding the cube							
11	**Order of Operations**	The order in which operations should be done	B I DM AS	Brackets Indices Divide and Multiply Add and Subtract					
12	≠	Not equal to							
13	**Decimal Place**	The number of digits after the decimal point							
14	**Rounding**	Changing a number to a simpler, easy to use value							
15	**Approximate**	Not exact but close to the true answer							
16	**Significant Figure**	The digits of a number that express a size to a particular degree of accuracy							
17	**Estimation**	A value that is close to the correct answer using a rough calculation							
18	**Factor**	A factor is a number that divides into another number.		Eg. factors of 6: 1, 2, 3 and 6					
19	**Multiple**	A multiple is a number from the times tables		Eg. the first five multiples of 4: 4, 8, 12, 16 and 20					
20	**Prime number**	A prime number is a number with exactly 2 factors							
		2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97							
21	**Product**	The answer when two or more numbers are multiplied together.							
22	**Index Form**	A number written to a power of an index							
23	**Dividing by a decimal**	Multiply both numbers by 10,100 etc until you have integers, then use bus stop							

24	**Prime factor decomposition**	Writing a number as a **product of its prime factors**	 Either way, the result is: $2 \times 2 \times 3 \times 5$ or $2^2 \times 3 \times 5$		
25	**Highest common factor**	HCF	The highest number that divides exactly into two or more numbers.	Eg. The HCF of 12 & 8 is 4	
26	**Lowest common multiple**	LCM	The smallest positive number that is a multiple of two or more numbers.	Eg. The LCM of 12 & 8 Is 24	

ROUNDING TO A SET NUMBER OF DECIMAL PLACES

27	- Count the number of decimal places you need. - Look at the number to the right of that digit 5 or more it rounds up 4 or less it rounds down			
28	e.g. 36.3486343	36.34|86343	To 2d.p. is 36.35	

ROUNDING TO SIGNIFICANT FIGURES

29	- Count the number of digits you need from the left (for numbers less than 1, zeros are not significant until after the first non-zero) - Look at the number to the right of that digit to decide if it rounds up or down - 5 or more it rounds up 4 or less it rounds down (for large numbers replace remaining digits with zeros as place holders)			
30	e.g. 324 627 938	32|4627938	To 2 sig.fig. is 320 000 000	
31	e.g. 0.0034792	0.0034|792	To 2 sig.fig. is 0.0035	

ESTIMATING

32	Round each number to 1 significant figure before doing any calculations				
33	e.g. $\frac{3.91 \times 8789.8}{620.9 \times 0.492}$	$\approx \frac{4 \times 9000}{600 \times 0.5}$	$\approx \frac{36000}{300}$	$\approx \frac{360}{3}$	≈ 120

RULES OF INDICES

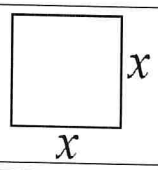
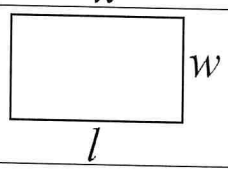
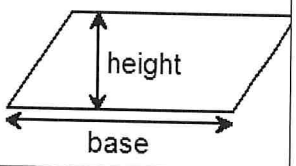
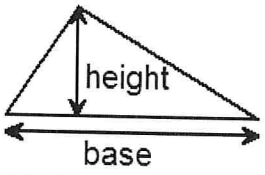
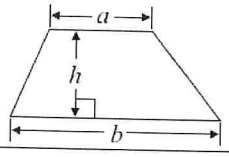
34	**Multiplying**	add the powers	$x^7 \times x^4 = x^{11}$
35	**Dividing**	subtract the powers	$x^9 \div x^3 = x^6$
36	**Brackets**	Multiply the powers	$(x^7)^2 = x^{14}$
37	**Power of 0**	Always = 1	$m^0 = 1$

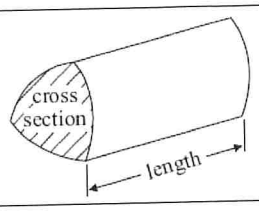
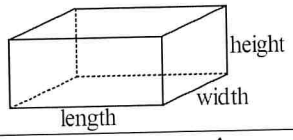
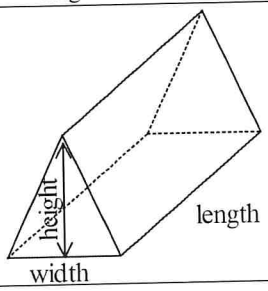
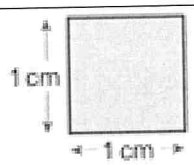
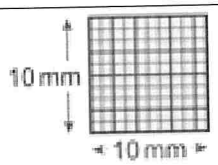
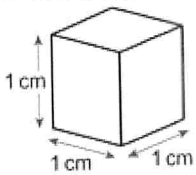
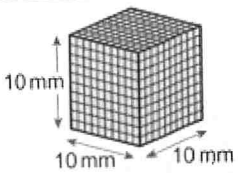
GCSE Mathematics Foundation
Perimeter, Area and Volume 1

DEFINITIONS

1	**Perimeter**	The distance around the outside of a 2D shape
2	**Area**	The space inside a 2D shape
3	**Volume**	The space inside a 3D shape
4	**Prism**	A 3D shape with a constant cross-section
5	**Isosceles Trapezium**	Has one line of symmetry
6		The two non-parallel sides are equal length
7	**Trapezia**	The plural of trapezium
8	**Capacity**	The amount of fluid a 3D object can hold
9	**Hectare**	An area of 10 000 m^2

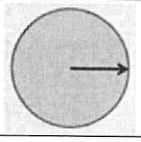
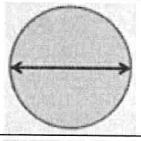
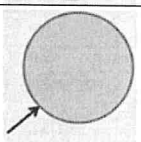
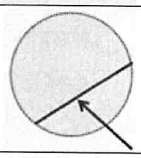
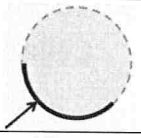
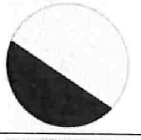
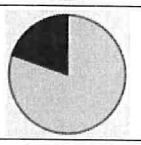
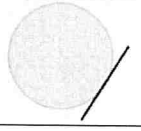
2D SHAPES

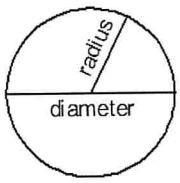
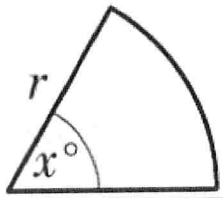
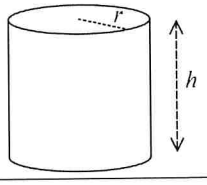
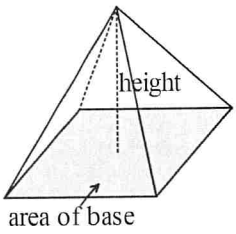
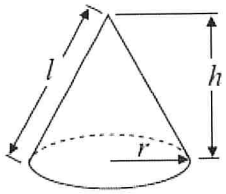
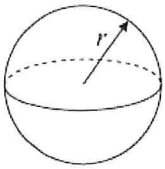
10	**Square**	$Area = length^2$	
11		$Perimeter = 4 \times length$	
12	**Rectangle**	$Area = length \times width$	
13		$Perimeter = 2 \times (length + width)$	
14	**Parallelogram**	$Area = base \times height$	
15	**Triangle**	$Area = \frac{1}{2} \times base \times height$	
16	**Trapezium**	$Area = \frac{1}{2}(a + b) \times h$	
17	**Compound Shape**	A shape made up of two or more simple shapes	
18		To find the area, split it into the simple shapes, find their areas and then add them together	

3D SHAPES			
19	**Surface area**	The total area of all the faces of an object	
20		You can use the shape's net to help calculate the surface area	
21	**Prism**	A 3D shape that has a constant cross-section through its length.	
22		$Volume = area\ of\ cross\ section \times length$	
23	**Cuboid**	$Volume = area\ of\ cross\ section \times length$ $Volume = length \times width \times height$	
24	**Triangular Prism**	$Volume = area\ of\ cross\ section \times length$ $Volume = \frac{1}{2} \times base \times height \times length$	
UNITS			
25	**Area units**	1 cm^2 = 100 mm^2	
26		1m^2 = 10000 cm^2	
27		 Area = 1 cm × 1 cm = 1 cm^2  Area = 10 mm × 10 mm = 100 mm^2	
28	**Volume units**	1 cm^3 = 1000 mm^3	
29		1 m^3 = 1000000 cm^3	
30		 Volume = 1 cm × 1 cm × 1 cm = 1 cm^3  Volume = 10 mm × 10 mm × 10 mm = 1000 mm^3	
31	**Capacity**	Is measured in *ml* and *litres*	
32		1cm^3 = 1*ml*	
33		1000cm^3 = 1 *litre*	

GCSE Mathematics Foundation
Perimeter, Area & Volume 2

DEFINITIONS

1	**Perimeter**	The distance around the outside of a 2D shape	
2	**Area**	The space inside a 2D shape	
3	**Surface Area**	The total area of all of the faces of a 3D shape	
4	**Volume**	The space inside a 3D shape	
5	**Radius**	Distance from the centre to the edge.	
6	**Radii**	Plural of radius	
7	**Diameter**	Distance from edge to edge passing through the centre.	
8	**Circumference**	Distance around the outside of a circle. (The perimeter)	
9	**Chord**	A line which touches the circumference at each end	
10	**Arc**	A part of the circumference of a circle	
11	**Segment**	A part of a circle between an arc and a chord	
12	**Sector**	A slice of a circle between an arc and two radii	
13	**Tangent**	A line outside a circle which only touches the circumference at one point	
14	**Semi-circle**	Half a circle	
15	**π (Pi)**	The constant ratio between the circumference and the diameter of circles.	3.14159265.....
16	**Formula**	Shows the relationship between different variables	
17	**Formulae**	Plural of Formula	
18	**Substitute**	Replace a variable with a number	

19	**Pyramid**	A 3D shape with a polygon as a base and triangular sides that meet at the top	
20	**Sphere**	A 3D shape where all the points on the surface are the same distance from the centre	
21	**Spherical**	Means in the shape of a sphere	
ERRORS IN MEASUREMENTS			
22	Measurements rounded to the nearest unit could be up to half a unit smaller or larger than the rounded value	If x is 3.4 correct to 1 dp The possible values of x are $3.35 \leq x < 3.45$	
IMPORTANT FORMULAE			
23	**Circumference of a circle**	$C = \pi d$	
24	**Area of a circle**	$A = \pi r^2$	
25	**Arc length**	$\frac{x}{360} \times 2\pi r$	
26	**Area of a sector**	$\frac{x}{360} \times \pi r^2$	
27	**Perimeter of a sector**	$\left(\frac{x}{360} \times 2\pi r\right) + 2r$	Arc length + 2 radii
28	**Cylinder**	$Volume = area\ of\ cross\ section \times length$ $Volume = \pi r^2 h$	
29		$Total\ Surface\ Area = 2\pi r^2 + 2\pi rh$	
30	**Volume of a Pyramid**	A 3D shape with a polygon as a base and triangular sides that meet at the top $Volume = \frac{1}{3} \times area\ of\ base \times height$	
31	**Cone**	$Volume = \frac{1}{3}\pi r^2 h$	
32		$Curved\ Surface\ Area = \pi rl$	
33		$Curved\ Surface\ Area = \pi r^2 + \pi rl$	
34	**Sphere**	$Volume = \frac{4}{3}\pi r^3$	
35		$Surface\ Area = 4\pi r^2$	

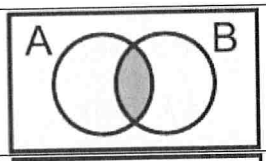
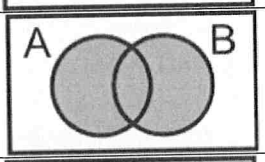
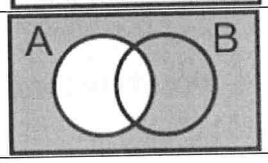
GCSE Mathematics Foundation
Probability

DEFINITIONS			
1	**Probability**	The extent to which an event is likely to occur	
2		It can be given as a fraction, a decimal, or a percentage	
3		It must take a value between zero and 1	
4	**Certain**	An event will definitely happen	e.g. The probability that the day after Sunday is a Monday is 1.
5		Probability of a certain event is 1	
6	**Impossible**	An event will definitely not happen	e.g. The probability that you will have your 3rd birthday tomorrow is 0
7		probability of an impossible event is 0	
8	**Biased**	Unfair	
9	**Unbiased**	Fair	
10	**Mutually Exclusive**	When events cannot happen at the same time	e.g. Rolling an even number and rolling a 5 on one dice
11	**Exhaustive**	Events are exhaustive if they cover all possible outcomes	
12	**Independent**	Two events are independent if the results of one do not affect the results of the other	
13	**Not Independent**	When the outcome of one event changes the probability of the next event, the two events are not independent	
14	**Sample space**	Shows all of the possible outcomes for one or more events	
15		Could be shown as a list, or a 2-way table, or a tree diagram	
THEORETICAL PROBABILITY			
16	**Calculating probabilities**	For equally likely outcomes the probability that an event will happen is $$P = \frac{number\ of\ successful\ outcomes}{total\ number\ of\ possible\ outcomes}$$	
17		e.g. The probability of rolling a 6 on a dice is $\frac{1}{6}$	
18		The probabilities of all possible outcomes add up to 1.	
19	If the probability that something WILL happen is **p** The probability that it WILL NOT happen is **1 - p**		e.g. Probability it will rain = 0.3 Probability it will not rain = 1 - 0.3 = 0.7
EXPERIMENTAL PROBABILITY			
20	Estimated probability	also known as experimental probability	
		$$estimated\ probability = \frac{frequency\ of\ event}{total\ frequency}$$	

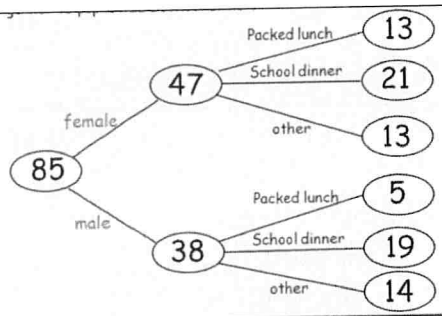
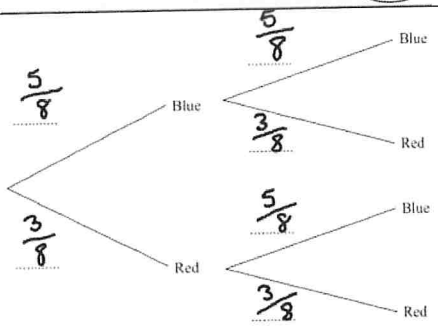
21	**Relative frequency**	In an experiment, how often something happens as a proportion of the number of trials
22		Can be used to estimate probabilities
23		Relative frequency = $\frac{how\ often\ something\ happens}{all\ outcomes}$
24	Your estimated probability gets more accurate the more trials you do	
25	**Predictions**	You can predict the number of outcomes you will get by: Predicted number of outcomes = probability × number of trials

VENN DIAGRAMS

26	{}	Curly brackets show a set of values	$5 \in \{\text{odd numbers}\}$ means "5 is in the set of odd numbers"
27	$\in$	Means "is an element of"	
28	**Element**	A "member" of a set	
29	ξ	Means the universal set	All the elements are being considered

30	**A ∩ B**	A intersection B	All elements in A AND B	
31	**A ∪ B**	A union B	All the elements in A OR B OR both	
32	**A '**	Not A	All the elements NOT in A	

TREE DIAGRAMS

33	**Frequency Tree**	Shows the number of different options for different choice	
34	**Probability Tree**	Shows all possible outcomes of an experiment	
35	**AND Rule**	Multiply the probabilities	P(A and B) = P(A) x P(B)
36	**OR Rule**	Add the probabilities	P(A or B) = P(A) + P(B)

GCSE Mathematics Foundation
Quadratic Equations & Graphs

DEFINITIONS

1	**Variable**	A letter representing a varying or unknown quantity	
2	**Coefficient**	A number which multiplies a variable.	e.g. 3 is the coefficient in 3y
3	**Term**	A part of an algebraic expression, could be a number, a variable or a product of both.	
4	**y-intercept**	The y value where a graph crossed the y – axis.	where x = 0
5	**x-intercept**	The x value where a graph crossed the x – axis.	where y = 0
6	**Index**	A small number to the upper right of a base number that shows how many times the base is multiplied by itself	
7	**Indices**	Plural of index	
8	**Quadratic**	An expression where the highest index is a 2.	e.g. $3x^2 + 5x + 4$
9	**Expand**	Means multiply out one or more brackets	
10	**"Show that"**	Means show your working	
11	**Function**	Describes a relationship between variables	
12	**Factorise**	Means writing an expression as a product of its factors The opposite of expanding out brackets	
13	**Product**	Means multiply	

EXPANDING DOUBLE BRACKETS

14 Multiply every term in the first bracket by every term in the second bracket

15

FOIL

e.g. $(x + 3)(x + 4)$

FIRST	x^2
OUTER	+4x
INNER	+ 3x
LAST	+ 12

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

GRID

e.g. $(x + 2)(x + 7)$

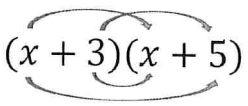
	x	+2
x	x^2	+2x
+7	+7x	+14

$= x^2 + 2x + 7x + 14$

$= x^2 + 9x + 14$

SMILEY FACE

e.g. $(x + 3)(x + 5)$



$x^2 + 3x + 5x + 15$

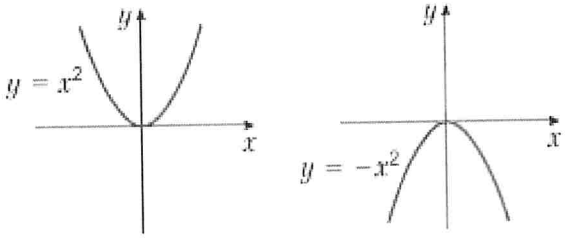
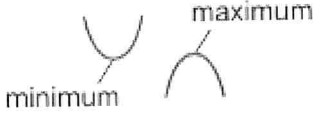
$= x^2 + 8x + 15$

16 To square a single bracket, multiply it by itself, then expand and simplify

e.g. $(x + 1)^2$

$= (x + 1)(x + 1)$

FACTORISING			
17	**Factorising a quadratic**	Multiply to 5 Factorise $x^2 + 5x + 6$ ← Add to 6 2 and 3 add to 5 2 and 3 multiply to 6 $(x + 2)(x + 3)$ Check: $(x + 2)(x + 3) = x^2 + 5x + 6$	
18	**Difference of two squares**	This is a quadratic equation with two squared terms, where one term is subtracted from the other	e.g. $x^2 - 81$ $x^2 - 81 = (x + 9)(x - 9)$

QUADRATIC GRAPHS			
19	The quadratic graph a curved shape called a parabola $y = ax^2 + bx + c$		
20	A positive x^2 term will give a ∪ shape		
21	A negative ($-x^2$) term will give a ∩ shape		
22	**Turning points**	The point where a curve turns in the opposite direction	
23		Either a maximum or a minimum point	
24	**Line of symmetry**	A quadratic graph will have a line of symmetry passing through its maximum or minimum point	
25	**Roots**	Solution to the quadratic equation $ax^2 + bx + c = 0$	
26		The x-values where the graph crosses the x-axis	
27		A quadratic equation can have 0, 1, or 2 solutions	

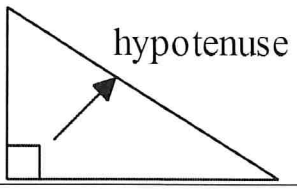
GCSE Mathematics Foundation
Ratio & Proportion

DEFINITIONS			
1	**Ratio**	A relationship between two or more quantities	
2	**Proportion**	Compares a part with the whole, i.e. fraction	
5	**Fraction**	Used to indicate a part of a whole number	
4	**Share**	Splitting into parts not necessarily equally but in a predefined ratio	
5	**Unit Ratio**	Used to compare ratios, one of the parts is 1	
		The only time it is permissible to have a decimal in a ratio	
6	**Proportional**	a change in one is always accompanied by a change in the other,	
7	**Unitary Method**	Find the value of 1 item, before multiplying to find the value of more	
		Used to work out which products give the better value for money	
8	**Exchange Rate**	The amount of money in a different currency that your currency will buy or sell for	
9	**Equivalent**	Ratios are equivalent if they have the same simplest form	
10	**Function**	a relation or expression involving one or more variables	
11	**Direct proportion**	When one variable changes the other changes in proportion	
12	**Inverse proportion**	When one variable increases the other decreases in proportion	
13	**Graphical**	Referring to a graph	
14	**Linear**	An equation that produces a straight line graph	
RATIO			
15	**Simplifying Ratio**	Divide each part of the ratio by a common factor	eg 12:18 simplifies to 2:3
16	**Simplest Form**	Divide each part of the ratio by the highest common factor, so it cannot reduce further	
		All parts must be integers	
17	**Share in a given ratio**	Divide an amount so the ratio of the final values simplifies to the given ratio.	Eg Divide £350 in the ratio 3:4 between Amy and Bob. 3+4 = 7 (There are 7 parts.) 350 ÷ 7 = 50 (Each part is worth 50) 3 x 50 = £150 for Amy 4 x 50 = £200 for Bob

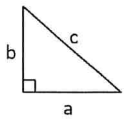
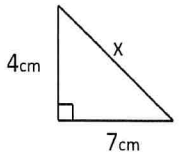
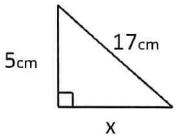
PROPORTION		
18	**Directly Proportional graph**	The graph of two quantities in direct proportion will go through the origin and have a positive gradient

GCSE Mathematics Foundation
Right-Angled Triangles

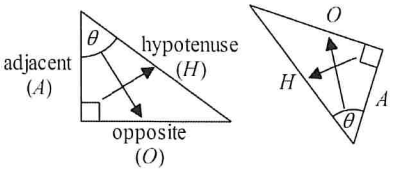
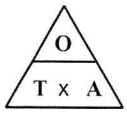
DEFINITIONS

1	**Hypotenuse**	The longest side of a right angled triangle. It is opposite the right angle	
2	**Substitute**	Replace a variable with a number	
3	**Surd**	A number written exactly using a square root	
4	**Surd Form**	"Giving an answer in surd form" means you do not need to work the square root out	

PYTHAGORAS' THEOREM

23	**Right-angled triangle**	A triangle that contains a right-angle.	
24	**Hypotenuse**	The side opposite the right-angle	
25	**Pythagoras' Theorem**  $a^2 + b^2 = c^2$	To find the hypotenuse  $X^2 = 4^2 + 7^2$ $X^2 = 16 + 49$ $X^2 = 65$ $X = \sqrt{65} = 8.06$ cm	To find a short side  $17^2 = x^2 + 5^2$ $289 = x^2 + 25$ $289 - 25 = x^2$ $x^2 = 264$ $x = \sqrt{264} = 16.25$cm

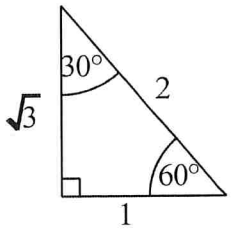
TRIGONOMETRY - SOHCAHTOA

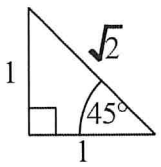
26	**Trigonometry**	The area of maths that studies of the relationships between the sides and angles of triangles		
27	**Labelling the triangle**	H = hypotenuse O = Opposite A = Adjacent θ is the angle involved		
28	**Sine**	$\sin\theta = \frac{O}{H}$	$\theta = \sin^{-1}\frac{O}{H}$	
29	**Cosine**	$\cos\theta = \frac{A}{H}$	$\theta = \cos^{-1}\frac{A}{H}$	
30	**Tangent**	$\tan\theta = \frac{O}{A}$	$\theta = \tan^{-1}\frac{O}{A}$	

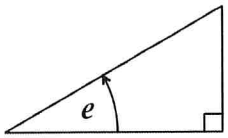
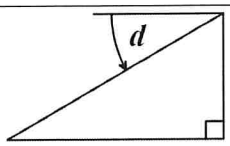
31	**Exact values**	

Θ	0°	30°	45°	60°	90°
Sin Θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos Θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan Θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

These can be found using the triangles:





32	**Angle of elevation**		Angle of depression	

GCSE Mathematics Foundation
Transformations

DEFINITIONS

1	**Transformation**	Altering a 2D shape in some way. Transformations are: - Rotation - Reflection - Translation or - Enlargement
2	**Rotation**	a "turning" movement of an image about a fixed point
3	**Reflection**	a "flipping" movement across a mirror line
4	**Translation**	a "sliding" movement of an image
5	**Enlargement**	a change in the size of the object (bigger or smaller)
6	**Scale factor**	The proportions by which the dimensions of an object will increase/decrease by
7	**Line of reflection**	a "mirror line" which is used to perform reflections
8	**Centre of rotation**	the fixed point used to rotate an object around
9	**Centre of enlargement**	a fixed point to enlarge an object from.
10	**Column Vector**	used to represent translations $\begin{pmatrix} x \\ y \end{pmatrix}$
11		x is the horizontal movement
12		x is the vertical movement
13	**Single Transformation**	Transform the image once
14	**Combination**	Transform the image multiple times
15	**Similarity**	images of the same shape, but in different sizes
16	**Congruent**	Same shape, same size
17	**Angle**	A measure of turn in degrees, measured with a protractor
18	**Direction**	Clockwise or anti-clockwise
19	**Co-ordinate**	Points on a graph (x, y)
20	**Origin**	The point $(0, 0)$, where the x and y axis cross
21	**Describe**	Write and verbally explain
22	**2D Shape**	One which has a length/height and a width, but no depth
23	**Object**	The shape given before a transformation occurs.
24	**Image**	The shape produced after the transformation

TRANSFORMATIONS			
23	**Rotation**	A "turning" movement of an image about a fixed point	
24		To describe, you need to write:	a) "Rotation" b) Angle of rotation c) Centre of rotation d) Direction of rotation
25	**Reflection**	A "flipping" movement across a mirror line	
26		To describe, you need to write:	a) "Reflection" b) The equation of the line of reflection
27	**Translation**	A "sliding" movement of an image	
28		To describe, you need to write:	a) "Translation" b) The column vector
29	**Enlargement**	A change in the size of the object (bigger or smaller)	
30		To describe, you need to write:	a) "Enlargement" b) The scale Factor c) The centre of enlargement