

Using letters to represent numbers

$5 + 5 + 5$	$y + y + y + y$	$20 - h$
3×5	$y \times 4$	$\frac{20}{h}$
5×3	$4 \times y$	$\frac{20}{h}$
	$4y$	

Addition and multiplication can be done in any order
 Commutative calculations

4 lots of 'y'

20 shared into 'h' number of groups

Equations with unknown on both sides

$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$

Keywords

- Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)
- Commutative: the order of the operations do not matter.
- Substitute: replace one variable with a number or new variable.
- Evaluate: work out

Substitution into expressions

$4y$ ← 4 lots of 'y'

If $y = 7$ this means the expression is asking for 4 'lots of 7'

4×7 OR $7 + 7 + 7 + 7$ OR $7 \times 4 = 28$

eg: $y - 2 = 7 - 2 = 5$

Solve equations with brackets

$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$+6 \quad +6$$

$$x = 3$$

- Simplify: grouping and combining similar terms
- Equivalent: something of equal value
- Coefficient: a number used to multiply a variable
- Solve: find a numerical value that satisfies an equation

Formulae and Equations

Formulae — all expressed in symbols

Substitute in values

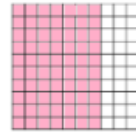
Equations — include numbers and can be solved

Convert FDP

$$\frac{70}{100}$$

This also means
70 ÷ 100

70 out of 100
squares
70 "hundredths"
= 7 "tenths"
0.7



70 hundredths
= 70%

Using a
calculator



S = D

Convert to a decimal

× 100 converts to a
percentage

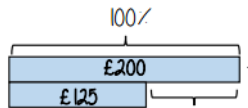
This will give you the answer
in the simplest form

Be careful of recurring decimals

e.g. $\frac{1}{3} = 0.3333333$
 $\frac{1}{3} = 0.\dot{3}$
 The dot above the 3

Percentage change

I bought a phone for £200.
A year later sold it for £125.



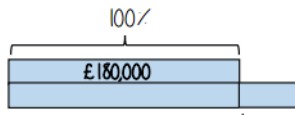
All values of change
compare to the
ORIGINAL value

Percentage loss

$$\frac{75}{200} \times 100 = 37.5\%$$

$$\frac{\text{Difference in value}}{\text{Original value}} \times 100$$

I bought a house for £180,000, I
later sold it for £216,000.



Percentage profit

$$\frac{36000}{180000} \times 100 = 20\%$$

Money made (profit value)

Keywords

Fraction: how many parts of a whole we have

Decimal: a number with a decimal point used to separate ones, tenths, hundredths etc.

Percentage: a proportion of a whole represented as a number between 0 and 100

Convert: change into an equivalent representation, often fraction to decimal to a percentage cycle.

Find the percentage of an amount (Calculator methods)



Using a multiplier

Find 65% of 80

Fraction, decimal, percentage conversion

$$65\% = \frac{65}{100} = 0.65$$

← The multiplier

$$0.65 \times 80 = 52$$

Using the percent button

Find 65% of 80

This brings up the % button on screen
You will see 65%

Type 65

Press **SHIFT** **(%)**

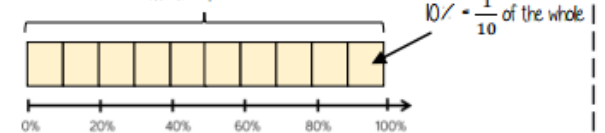
Press **80** and then press =

You can also use the
calculator to support non
calculator methods and
find 1/ or 10/ then add
percentages together

"of" can represent "x" in calculator methods

Find the percentage of an amount (Mental methods)

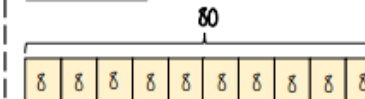
The whole represents 100%



$$10\% = \frac{1}{10} \text{ of the whole} \quad 50\% = \frac{5}{10} = \frac{1}{2} \text{ of the whole}$$

$$20\% = \frac{2}{10} = \frac{1}{5} \text{ of the whole} \quad 5\% = \frac{1}{20} \text{ of the whole}$$

Find 65% of 80



Method 1:
 $65\% = 10\% \times 6 + 5\%$
 $= (8 \times 6) + 4$
 $= 52$

Method 2:
 $65\% = 50\% + 10\% + 5\%$
 $= 40 + 8 + 4$
 $= 52$

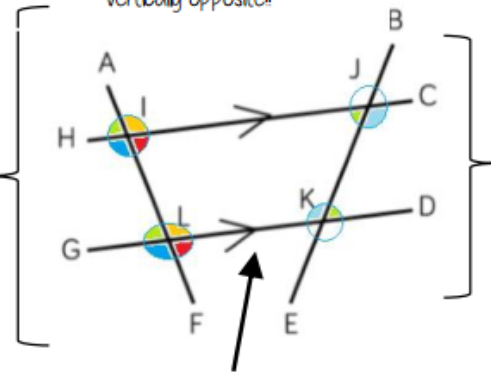
For bigger percentages it is sometimes easier to take away from 100%

Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!

Lines OF and BE are transversals (lines that bisect the parallel lines)

Corresponding angles often identified by their 'F shape' in position.



Alternate angles often identified by their 'Z shape' in position

This notation identifies parallel lines

Basic angle rules and notation



The letter in the middle is the angle
The arc represents the part of the angle



Acute Angles
 $0^\circ < \text{angle} < 90^\circ$



Obtuse
 $90^\circ < \text{angle} < 180^\circ$

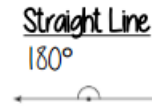


Reflex
 $180^\circ < \text{angle} < 360^\circ$



Right Angles
 90°

Right angle notation



Straight Line
 180°



Angle Notation: three letters ABC
This is the angle at B = 113°
Line Notation: two letters EC
The line that joins E to C.



Vertically opposite angles
Equal
Angles around a point
 360°

Properties of Quadrilaterals



Square
All sides equal size
All angles 90°
Opposite sides are parallel



Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles



Rectangle
All angles 90°
Opposite sides are parallel



Trapezium
One pair of parallel lines

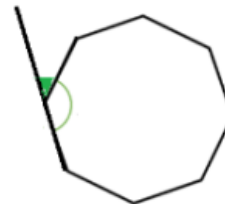


Rhombus
All sides equal size
Opposite angles are equal



Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Missing angles in regular polygons



Exterior angle = $360 \div 8 = 45^\circ$

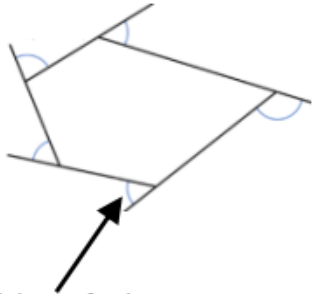
Interior angle = $\frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8} = 135^\circ$

Exterior angles in regular polygons = $360^\circ \div \text{number of sides}$

Interior angles in regular polygons = $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

Sum of exterior angles

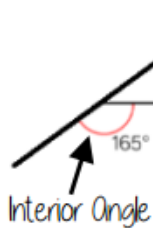
Exterior angles all add up to 360°



Exterior Angles

Are the angle formed from the straight-line extension at the side of the shape

Using exterior angles



Exterior Angle

Interior angle + Exterior angle = straight line = 180°
Exterior angle = $180 - 165 = 15^\circ$

Number of sides = $360^\circ \div$ exterior angle
Number of sides = $360 \div 15 = 24$ sides

Keywords

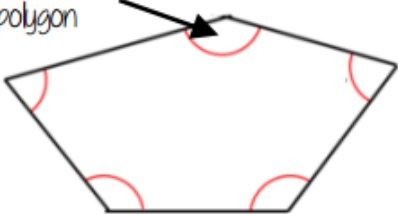
- Polygon: A 2D shape made with straight lines
- Scalene triangle: a triangle with all different sides and angles
- Isosceles triangle: a triangle with two angles the same size and two sides the same size
- Right-angled triangle: a triangle with a right angle
- Regular polygon: All the sides have equal length; all the interior angles have equal size

Sum of interior angles

$(\text{number of sides} - 2) \times 180$

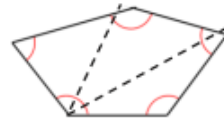
Interior Angles

The angles enclosed by the polygon



This is an **irregular** polygon
— the sides and angles are different sizes

Sum of the interior angles = $(5 - 2) \times 180$

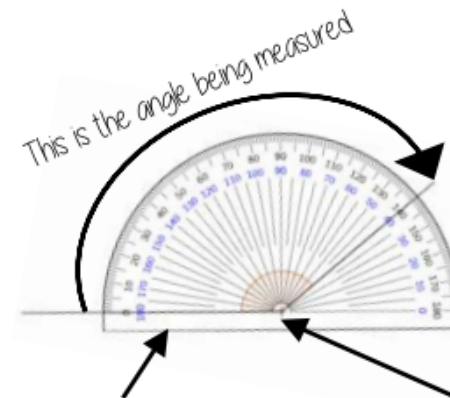


This shape can be made from three triangles
Each triangle has 180°

Sum of the interior angles = 3×180
= 540°

Remember this is **all** of the interior angles added together

Measure angles to 180°



The base line follows the line segment

Make sure the cross is at the point the two lines meet

Read from 0° on the base line.
Remember to use estimation.
This is an obtuse angle so between 90° and 180°

Add/Subtraction any fractions

$$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

Add/Subtraction fractions (improper and mixed)

$$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = \frac{22}{10} - \frac{13}{10} = \frac{9}{10}$$

- Convert to an improper fraction
- Calculate with common denominator

Dividing any fractions *Remember to use reciprocals*

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{8}{15}$$

Multiplying by a reciprocal gives the same outcome

Represented

Partitioning method

$$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$$

Multiplying non-unit fractions

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Shade in 3 parts

Repeat it on this many rows

This many columns

This many rows

Modelled:

Parts shaded

Total number of parts in the diagram

Keywords

Integer: a whole number that is positive or negative

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts

Mixed numbers: a number with an integer and a proper fraction

Improper fractions: a fraction with a bigger numerator than denominator

Negative: a value less than zero.

Commutative: changing the order of the operations does not change the result

Estimation

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

$$210 + 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach $300 + 900$.

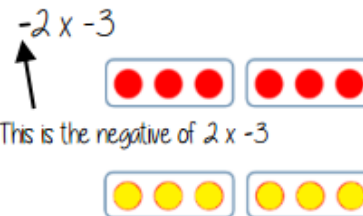
The correct estimation would be $200 + 900 = 1100$.

Multiply/ Divide directed numbers



Two representations of the same calculation $2 \times -3 = -6$

Negative, Negative calculation

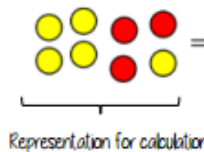


This is the negative of 2×-3

$$-2 \times -3 = 6$$

The act of making counters into their negative is turning them over

Subtract directed numbers

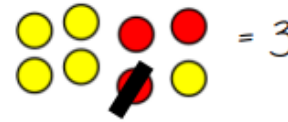


2 "Subtract" – means take away or remove

$$2 - -1 = 3$$

Take away one

Start with the representation of 2



$$2 - -3 = 5$$



Generalisation

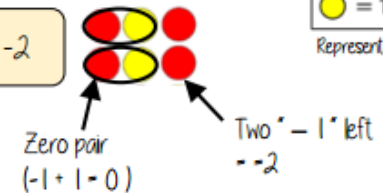
$$- - + = +$$

Divisions are the inverse operations

Add directed numbers



$$2 + -4 = -2$$



Zero pair $(-1 + 1 = 0)$

Two "– 1" left $- -2$

$$8 + -3 = 5$$



Partitioning

$$8 + -3 = 5$$

$$5 + 3 + -3 = 5$$

Partition the value to create a zero pair calculation

Generalisation

$$+ - = -$$