

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets

$3(2x + 4) = 30$

Expand the brackets

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$-6 \quad -6 \quad x = 3$$

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

$$x > 3$$

Inequalities with negatives

Method 1: Make x positive first

$$2 - 3x > 17$$

$$+3x \quad +3x$$

$$2 > 17 + 3x$$

$$-17 \quad -17$$

$$-15 > 3x$$

$$\div 3 \quad \div 3$$

$$-5 > x$$

x is true for any value smaller than -5

✓ CHECK IT!
 $2 - 3(-6) = 20$
 TRUE/ CORRECT

Smaller ← | | | | | → Bigger
 -7 -5 -3

Equations with unknown on both sides

$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$

Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$$5(x + 4) < 3(x + 2)$$

$$5x + 20 < 3x + 6$$

$$2x + 20 < 6$$

$$2x < -14$$

$$x < -7$$

Check it!
 $5(-8 + 4) < 3(-8 + 2)$
 $5(-4) < 3(-6)$
 $-20 < -18$
 ✓ -20 IS smaller than -18

Method 2: Keep the negative x

$$2 - 3x > 17$$

$$-2 \quad -2$$

$$-3x > 15$$

$$\div -3 \quad \div -3$$

$$x > -5$$

x is true for any value bigger than -5

This cannot be true...

When you multiply or divide x by a negative you need to reverse the inequality

$$x < -5$$

Formulae and Equations

Formulae – all expressed in symbols

Substitute in values

Equations – include numbers and can be solved

Rearranging Formulae (one step)

x	z
y	

$$x = y + z$$

Rearrange to make y the subject

$$y = x - z$$

$$y \rightarrow +z \rightarrow x$$

$$y \leftarrow -z \leftarrow x$$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution

Language of rearranging....

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$\div 4 \quad \div 4$$

$$x = 3$$

In a formula (make x the subject)

$$xy - s = a$$

$$+s \quad +s$$

$$xy = a + s$$

$$\div y \quad \div y$$

$$x = \frac{a + s}{y}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first

$$y = \frac{4x + 9}{2}$$

Gradient = $\frac{4}{2} = 2$

What do I need to be able to do?

By the end of this unit you should be able to:

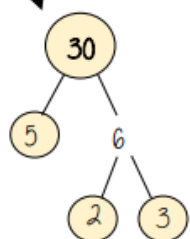
- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

Keywords

- Multiples:** found by multiplying any number by positive integers
- Factor:** integers that multiply together to get another number.
- Prime:** an integer with only 2 factors.
- HCF:** highest common factor (biggest factor two or more numbers share)
- LCM:** lowest common multiple (the first time the times table of two or more numbers match)
- Verify:** the process of making sure a solution is correct
- Proof:** logical mathematical arguments used to show the truth of a statement
- Binomial:** a polynomial with two terms
- Quadratic:** a polynomial with four terms (often simplified to three terms)

Factors, Multiples and Primes

Multiplication part-whole models



All three prime factor trees represent the same decomposition

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share

R

True or False?

Conjecture

A pattern that is noticed for many cases

1, 2, 4, ...
The numbers in the sequence are doubling each time.

Counterexamples



This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Always, Sometimes, Never true.

Always Every value always supports the statement

Sometimes Examples show the statement being true and counter examples to show when it is false.

Never No example supports the statement

Examples to try

- 0 and 1
- Fractions
- Negative numbers

Show that

Numerical verification

Show the stages to a solution with numerical values

Algebraic verification

Show algebraic properties of the solution
You may want to use pictorial images to support this

Proof

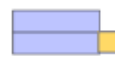
Simple proofs using algebra

Compare the left hand side of an equation with the right hand side – are they the same or different?

Conjectures



Even
(2n)
Multiple of 2



Odd
(2n + 1)
One more than any even

Use numerical verification first
Use pictorial verification – the representations of numbers of odd and even

Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

Positions of numbers in relation to n form expressions.
Eg one space to the right of n
n + 1

Eg One row below n
n + 10

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

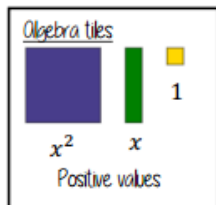
The size of the grid for generalisation changes the relationship statements

Expanding binomials

$$2(x + 2) \equiv 2x + 4$$



Algebra tiles can represent a binomial expansion
Has two terms



$$(x + 3)(x + 3) \equiv x^2 + 6x + 9$$



This is a quadratic.
It has four terms which simplified to three terms

The order of the binomial has no impact on the outcome.
eg (x + 3)(3 + x)